SUPPLEMENTAL MATERIAL

for

The Weather Research and Forecasting Model with Aerosol Cloud Interactions (WRF-ACI): Development, Evaluation, and Initial Application

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Introduction: The first part of this supplementary material contains detailed descriptions of the multi-scale Kain-Fritsch (MSKF) scheme while the second part contains additional discussions and figures in support of results presented in the paper. Table of contents is given below.

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TEXT S1: Descriptions of the Multi-Scale Kain-Fritsch (MSKF) Scheme

The MSKF is based on the Kain-Fritsch (Kain 2004) scheme which was originally developed for use in regional models and is suitable at horizontal grid spacing of about 25 km. As the computational resources are increasingly available, regional models are being routinely used in the so-called grey zone scales (i.e., 12 to 1 km). One of the major issues with such grid spacings is the lack of scale-awareness in many cumulus parameterization schemes (CPSs) where the role of a CPS should become increasingly less important as grid spacing decreases in such a way that there is a gradual handover of the convection treatment to a grid scale microphysical scheme. Thus, at about kilometer or sub-kilometer grid resolutions a CPS will not have any role, leaving the responsibility to restore stability to the atmosphere solely to a grid scale microphysical scheme. In a visionary work by Arakawa et al. (2011), two approaches were presented to develop a unified treatment of convection for use in multiscale modeling. In that work an argument was presented that a single cloud physics that satisfies the needs of a coarse grid model (e.g., 250 km grid spacing global climate model) as well as a cloud resolving model (e.g., 1 km grid spacing) can be developed. At coarse grid spacing, convective cloud fraction ($\sigma_c$) needs to be very small as compared to the area of a coarse grid cell and also mean vertical velocity of environmental air ($W_e$) will be far smaller than that of the convective updraft velocity($W_c$). As the grid spacing approaches cloud resolving scales, $\sigma_c$ tends to become 1, and $W_c$ vanishes since $W_e$ assumes the magnitudes of $W_c$.

Until such a unified convection scheme is developed, our choices are limited to modifying the existing CPSs such that scale-awareness can be implemented into these schemes in the hopes that these scale-aware CPSs will gradually handover the stability restoration responsibility to a grid scale microphysical scheme as grid spacing decreases. This approach flies against the concept of a unified multiscale convection theme proposed by Arakawa et al. (2011), but it is a necessary tool that needs to be developed as an interim measure for weather and climate research. One such tool has been in development, namely the multi-scale Kain-Fritsch (MSKF) parameterization scheme, and is based on the theme that $\sigma_c << 1$ at all grid spacings including the grey zone scales (e.g., ~12 to ~1 km).

Since there is no single paper that describes all features of the MSKF scheme, here we briefly present details of the MSKF scheme. For greater details readers are referred to our earlier papers. To begin, we considered the cloud resolving modeling results of Arakawa et al. (2011) that present convective cloud fraction (Fig. 8), $\sigma_c$, at 3 km altitude averaged over all sub-domains with varying sizes from 2 km to 512 km sub-domains, referred to as “grid spacings.” The variation of $\sigma_c$ across these grid spacings indicated that until up to 32 km grid spacing $\sigma_c << 1$, and then it gradually increases to one at 2 km grid spacing, with a bimodal distribution. In the MSKF a scaling parameter, $\beta$, has been introduced that follows a very similar bimodal variation across various spatial scales. This $\beta$ parameter is used to introduce scale awareness to various convection parameters used in the MSKF scheme in a way that controls MSKF’s ability to
remove convective instability such that MSKF gradually becomes unimportant at 1 km or sub-kilometer grid spacing. This process ensures the gradual handing over of the responsibility of stability restoration to a grid scale cloud microphysical scheme. The concept is shown as a schematic in below shown first figure.

The question is: at a given grid spacing, which cloud physics, i.e., subgrid scale (MSKF) or grid scale (GSCM), should restore the stability to the atmosphere? So, at a coarse grid spacing, e.g., 25 km, depending upon the thermodynamic state of the modeled atmosphere, either MSKF or GSCM will restore the stability to the atmosphere. At that grid spacing, convection cannot be resolved, and thus a CP scheme is needed for warm periods while GSCM can alone can handle the grid scale saturation that typically occurs in cool periods. As grid spacing decreases to about 1 km, then most types of diverse cumulus convections are resolved (Kajikawa et al., 2016); thus, there is no need for the MSKF at such fine grid spacing; thus, GSCM alone will restore the stability to the atmosphere irrespective of season.

The $\beta$ parameter, referred to as scale factor and it is estimated as:

$$\beta = 1 + \ln \left( \frac{25}{\Delta x} \right)$$

(E1)

and its variation across different grid spacings is shown in second figure. Note that for any grid spacing greater than 25 km, $\beta$ assumes the value of one as the original Kain-Fritsch scheme was developed for use at about 25 km grid spacing.

Schematic for scale dependency of subgrid scale (MSKF) and grid scale cloud microphysical (GSCM) schemes. MSKF=Multi-Scale Kain-Fritsch scheme; GSCM=Grid scale Cloud Microphysics scheme.
Variation of $\beta$ parameter used in the MSK scheme (Zheng et al., 2016).

The $\beta$ parameter plays an important role in the MSKF to provide a scale-awareness to many of the formulations used to represent moist convection processes. Since each component of the developments of the MSKF was documented in different papers and conference presentations, for easy access we present here various components of the MSKF scheme. These components (a) to (g) are shown in below figure and these are described in the following subsection.

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Various convective formulations introduced to develop the MSKF scheme available in the WRF4.0

Incorporation of Convective Cloud and Radiation Interactions

In many regional CPSs, simulated convective clouds did not interact with radiation and thus cloud-radiation interactions and their impacts were absent in such schemes. Basically, they all lacked cloud fraction representations and thus these schemes precipitated in clear skies rather than cloudy skies, while explicitly simulated or resolved clouds were allowed to impact radiation. To alleviate this deficiency in the Kain-Fritsch scheme, Alapaty et al. (2012) introduced convective cloud radiation interactions as the first modification to the development of the MSKF scheme. They have incorporated this key feedback process into MSKF and a radiation scheme in the Weather Research and Forecasting model and evaluated the impacts of this process in short-term weather and multiyear climate simulations. Introducing subgrid scale convective cloud-radiation feedbacks led to a more realistic simulation of attenuation of downward surface shortwave radiation. Reduced surface shortwave radiation moderated the surface forcing for convection and resulted in a notable reduction in precipitation biases. A more in-depth consideration of the effects of subgrid scale clouds regional climate simulations and associated feedbacks were documented in Herwehe et al. (2014).

In the original WRF (version 3.3.1) model, grid scale cloudiness is estimated in a two-stage process. First, if a grid cell is saturated (with respect to water or ice) then that grid cell is assigned 100% cloudiness. Otherwise, that grid cell is assigned 0% cloudiness. Then, the impacts of other physical and dynamical processes (such as cumulus detrainment, 3-D advection, etc.)
the grid scale saturation alter the saturation value. This modified saturation value for each grid cell is then utilized to re-estimate partial grid scale cloudiness using an empirical formulation. Thus, modified grid scale cloudiness can vary anywhere between 0 to 100% instead of being simply set to binary values. Our analysis has indicated that the modified grid scale cloudiness still hovers close to either 0% or 100%. This modified grid scale cloudiness is then taken as input for our research in the estimation of total cloudiness due to all clouds. The subgrid scale cumulus cloudiness formulation used in the Community Atmosphere Model version 5 (CAM5) (Neale et al., 2010), originally suggested by Xu and Krueger (1991), is selected for implementation into the MSKF scheme in the WRF model. Following the CAM5 methodology, MSKF cloud updraft mass fluxes are used to estimate the fractional three-dimensional cloudiness associated with shallow and deep cumulus clouds. Since convection is penetrative, it can punch through the existing grid scale clouds. Also, subsidence associated with convection will affect the grid scale saturation leading to reduction/dissipation of existing grid scale clouds. The CAM5 formulation accounts for these two types of convection impacts on the grid scale cloudiness. Finally, grid scale cloudiness is further modified to ensure that the total cloudiness composed of contributions from grid scale and subgrid scale clouds cannot exceed 100%. To maintain consistency, we also adjust grid scale condensates according to changes made to the grid scale cloudiness. The standard WRF considers cloudiness only from the grid scale clouds and associated liquid and ice water paths in radiative transfer calculations. However, to include the radiative contributions by the convective clouds, liquid and ice water condensates associated with the KF subgrid clouds are added to corresponding adjusted grid scale condensates. Finally, total liquid and ice water paths and cloudiness values for all clouds are then used in the Rapid Radiative Transfer Model for global (RRTMG) models (Iacono et al., 2008) to affect the shortwave and longwave radiative processes. Thus, the modified RRTMG used in the study considers radiative effects of grid scale as well as subgrid scale clouds consistent with respective cloud physical formulations.

The set of equations used to represent the subgrid scale cumulus cloud and radiation interactions are same as those used in the Community Atmosphere Model (version 5) (Neale et al., 2010). If \( M_u^d \) represents in-cloud updraft mass flux estimated by the MSKF for deep convective clouds, then the cloudiness \( \sigma_{cd} \) associated with subgrid scale deep convective cumulus clouds can be written as:

\[
\sigma_{cd} = k_1 \log e (1 + k_2 M_u^d)
\]

where the constants \( k_1 \) and \( k_2 \) are specified as 0.14 and 500.0, respectively. Similarly, cloudiness \( \sigma_{cs} \) associated with subgrid scale shallow convective cumulus clouds can be written as (Collins et al., 2006):

\[
\sigma_{cs} = k_3 \log e (1 + k_4 M_u^s)
\]

where \( M_u^s \) is updraft mass flux in a shallow convective cloud, and \( k_3 \) and \( k_4 \) are constants (0.07 and 500.0). Then, the final cloudiness for the deep \( \sigma_{cd} \) and shallow \( \sigma_{cs} \) clouds is limited to
60% and 20%, and the total cumulus cloudiness that is not allowed to exceed 80% of the grid volume is written as:

\[ \sigma_c = \sigma_{cd} + \sigma_{cs} \leq 0.8 \]

Note that this upper limit (0.8) was set as a precautionary numerical constraint while simulated summer seasonal (JJA 1989) values indicated that the total (\( \sigma_c \)) never exceeded 0.1 in a multi-year simulation (Herwehe et al., 2014) satisfying the condition, \( \sigma_c \ll 1 \), mentioned earlier. Since cumulus convection is penetrative, it can punch through the existing grid scale clouds. Also, subsidence associated with convection will affect the grid scale saturation leading to reduction/dissipation of existing grid scale clouds. The Neale et al. (2010) formulation accounts for these two types of convection impacts on the grid scale cloudiness. Thus, the grid scale cloudiness (\( \sigma_g \)) computed by a grid scale cloud microphysical formulation is further modified as \( \sigma_{gf} \) to account for the above described convection impacts on the grid scale clouds, and it is written as:

\[ \sigma_{gf} = (1 - \sigma_c) \sigma_g \]

Further, it is ensured that the total cloudiness (\( \sigma_t \)) composed of contributions from grid scale and subgrid scale clouds cannot exceed 100%, and it can be written as:

\[ \sigma_t = \sigma_c + \sigma_{gf} \leq 1 \]

Since the grid scale cloudiness is altered by an amount, \( \Delta \sigma_g = \sigma_g - \sigma_{gf} \), to maintain the consistency the grid scale condensate (\( q_g \)) is also adjusted accordingly as:

\[ Q_g = (1 - \Delta \sigma_g) q_g \]

reflecting changes in the grid scale cloudiness. All these parameters are then used in the radiative transfer model to account for the impacts of subgrid scale cloudiness on the atmospheric shortwave and longwave radiation.

Dynamic Convective Adjustment Timescale

In many CPSs (e.g., Betts, 1986; Kain, 2004) the convective adjustment timescale (\( \tau \)) plays an important role in determining drying and heating associated with subgrid scale precipitation, and it represents the time needed to restore stability to the atmosphere by removing convective instability (e.g., Bechtold et al., 2008; Mishra and Srinivasan, 2010; Yang et al., 2013; Bullock et al., 2015). Done et al. (2006) found that varying \( \tau \) from minutes to one day resulted in changing all convective parameterization generated subgrid scale precipitation to grid scale precipitation. Yang et al. (2013) found that when varying \( \tau \) from 1800 to 28,800 s, the frequency of generating good global model simulations with \( \tau > 12000 \) was much smaller than when \( \tau \) was less than 12,000 s, indicating how variability in \( \tau \) impacted climate simulations. Adames (2017) and Wang et al. (2017) found that using a spatially varying \( \tau \) has resulted in a spatial distribution of surface
precipitation for MJO that was largely in agreement with satellite-derived precipitation distributions and better simulation of MJO. In a global modeling study by separating and analyzing grid- and subgrid scale precipitation from total precipitation, Kooperman et al. (2018) found that there exists a deficiency in that a global model was unable to capture the intensity of moderate precipitation rates and was attributed to limitations associated with a convective parameterization. In addition to these studies, our prior research also indicated that hyperactivity of a CPS (i.e., excess triggering of the CPS) is primarily due to the uncertainty in \( \tau \) specification although other modeling factors may also contribute to the hyperactivity of CPSs. In a majority of CPSs used in weather, climate, hydrological, and air pollution studies, the \( \tau \) has been used as a global constant and is often tuned according to the needs of a specific study. Thus, literature indicated that the simulated precipitation can be largely influenced by uncertainty in \( \tau \) estimations.

A new scale-aware and generalized convective adjustment timescale, \( \tau \), was developed by He and Alapaty (2018), and its development is described here. The convective adjustment timescale (\( \tau \)) is the time required to restore stability to the atmosphere, and it depends on the thermodynamic structure of both the subcloud layer and the cloud layers. Thus, we first partition the troposphere into two layers: (1) the subcloud layer and (2) the cloud layer. For each of these layers we propose a depth-averaged velocity and length (depth) scale, which are relevant to both shallow and deep convective clouds. Further, we use vertical scales in developing a generalized formulation that works for both the shallow and deep convective clouds.

The length scales for the subcloud layer and cloud layer are readily available from any boundary layer and convective parameterization scheme, and can be defined as the height of the cloud base (i.e., lifting condensation level, LCL) above the land/water surface and the distance between the cloud base and the equilibrium level of a convective cloud, respectively. To fully develop the new stability restoration method for the estimation of \( \tau \), we must also estimate the velocity scales for the subcloud and cloud layers, which is the closure problem. The cloud work function (Arakawa and Schubert, 1974), \( C_{wf} \), for convective cumulus clouds can be written as:

\[
C_{wf} = \int_{z_a}^{z_T} \frac{dK_u}{dt} \frac{dz}{dz} = \int_{z_a}^{z_T} \frac{g}{\theta_v} m_u (\theta_{vc} - \theta_v) dz
\]

where \( z_T \) and \( z_B \) are the altitudes of cloud top and base, \( g \) is the acceleration due to gravity, \( \theta_v \) and \( \theta_{vc} \) are the virtual potential temperatures of the environment and convective cloud, respectively, \( m_u \) is the updraft mass flux, \( dz \) is the atmospheric layer thickness, and \( K_u \) is the convective kinetic energy of an updraft. Thus, \( C_{wf} \) is an integral measure of the buoyancy force governing convective kinetic energy generation. If \( m_u \) were truly a constant within a convective cloud, then the right-hand side of the above equation is equal to the product of \( m_u \) and entrained convective available potential energy, \( A_v \). However, \( m_u \) is not a constant within a convective cloud, and thus we approximate the above equation by using a vertical mean of updraft mass flux as \( \bar{m} \). Then, \( \bar{m} \) becomes independent of altitude within a cloud, and the above can be re-written as:

\[
C_{wf} \approx \bar{m} A_v
\]
To help estimate $\bar{m}$ we have analyzed updraft mass flux profiles from a bulk plume scheme (Lawrence and Rasch, 2005) and large eddy simulations (Kuang and Bretherton, 2006; Grabowski et al., 2006). We found that the cloud base mass flux, $m_b$, is related to $\bar{m}$ by a one-dimensional parameter, $\delta$, that varies from about 0.75 to 1.2 for various temporal evolutions of simulated deep clouds. This leads to a simple relation, $\delta = \frac{m}{m_b}$, which allows the above equation to be written as:

$$C_{wf} = \bar{m} A_e = \delta m_b A_e = \int_{z_b}^{z_e} \frac{dK_u}{dt} dz = \int_{z_b}^{z_e} w_u \frac{\partial w^2}{\partial z} dz \approx W_{CL}^3$$

where $W_u$ is the mean updraft velocity, and $W_{CL}$ is cloud-averaged velocity scale for shallow or deep clouds. Note that the non-linear integration of the above equation becomes repetitive and thus needs a truncation of higher-order terms. Then, the above equation can be rewritten for $W_{CL}$ as:

$$W_{CL} = (\delta m_b A_e)^{1/3}$$

If $\delta$ is set to unity, then the above equation is exactly the same as that formulated by Grant and Lock (2004) for shallow convective clouds, which was derived from using subcloud layer turbulent kinetic energy budgets. Since the cloud work function concept is applicable across a spectrum of convective clouds, the above equation can be used for both shallow and deep convective clouds by setting appropriate values for $\delta$. Thus, the $\tau$ can be estimated as (Bechtold et al., 2008; Zheng et al., 2016):

$$\tau = \frac{H}{W_{CL}} \beta = \frac{H}{(\delta m_b A_e)^{1/3}} \beta$$

where the scale factor $\beta$ is estimated as $\beta = 1 + \ln \left( \frac{25}{\Delta x} \right)$, and $H$ is depth of a convective cloud. For more details of this formulation, we refer readers to our earlier work (Zheng et al., 2016).

The $\tau$ formulation shown above has been found to work well at various horizontal grid spacings (Zheng et al., 2016). However, its applicability is limited because it can be only used with mass flux CPSs that use $m_b$. To generalize this formulation such that it can also be used in any CPS such as non-mass flux schemes (e.g., Betts, 1986), we follow Grant (2001) who suggested that $m_b$ can be proportional to the subcloud layer velocity ($W_{sb}$). As $W_{sb}$ can be used in both mass flux and non-mass flux CPSs, a generalized formulation that incorporates cloud and subcloud layer interactions is proposed here as:
\[ \tau = \frac{H}{(\alpha \delta W_{sb} A_e)^{1/3}} \beta \]  

(E1)

where \( \alpha \) is a non-dimensional Tokioka parameter equal to 0.03 (Tokioka et al., 1988) and \( \delta \) is set to be 1 for all convective clouds in this work. Adapting the formulation suggested by Alapaty and Alapaty (2001) for convective conditions in boundary layer, \( W_{sb} \) can be written as:

\[ W_{sb} = \sqrt{3.8u_*^2 + 0.22W_c^{1/2} + 1.9u_*^2 \left( \frac{-z_{LCL}}{L} \right)^{2/3}} \]

where \( u_* \) is the surface friction velocity, \( W_c^* \) is the convective velocity in the boundary layer, \( z_{LCL} \) is height of the LCL, and \( L \) is the Monin-Obukhov length. For stable boundary layer conditions, \( W_{sb} \) can be written as:

\[ W_{sb} = \sqrt{3.8u_*^2} \]

To summarize, the new stability restoration method shown in Eq. E1 can be used to estimate \( \tau \) in both mass flux and non-mass flux CPSs and applied to both shallow and deep convective clouds. All parameters on the right-hand side of Eq. E1 can be readily estimated or diagnosed, even if such parameters are not used in a particular CPS. Advantages of our new stability restoration method over our old formulations (Bullock et al., 2015; Zheng et al., 2016) are that the \( \tau \) estimation is directly dependent on the thermodynamics of the subcloud layer and convective cloud macrophysical parameters.

Precipitation Double Counting Elimination

In many regional and global models, at coarser resolutions where a CPS is used to estimate the unresolved convective clouds and associated precipitation, double counting of precipitation can occur due to the state of the modeled atmosphere when saturation occurs at grid scale. Since a CPS can trigger convection irrespective of atmospheric saturation, both the grid scale and subgrid scale CPS can be triggered for a saturated atmospheric column producing precipitation from grid and subgrid scale cloud parameterizations. This leads to the so-called double counting of precipitation. This situation can occur at a grey zone scale (~12 to 1 km) grid spacing, particularly during warm periods. This double counting can lead to overestimation of precipitation as compared to the measurements. To avoid the double counting, in the MSKF scheme the saturation state of each grid column is first checked by estimating integrated/averaged relative humidity of atmospheric layers (with respect to water as well as ice as appropriate) between convective cloud base and top using methods suggested by Bolton (1980). If the averaged value exceeds saturation value (100%), then that grid column does not need a CPS since the cloud layers are saturated or supersaturated, and thus MSKF drops out without estimating convective precipitation and impacting the thermodynamic profiles. Then, the grid scale cloud microphysics formulation will act on that grid column removing convective instability and estimating resolved precipitation. This way, at any time during a simulation, either
MSKF CPS or a grid scale cloud formulation will accomplish the restoration of the stability to the atmosphere and thereby eliminating double counting.

Double-Moment Convective Microphysics

In many CPSs temperature and humidity profiles are adjusted to account for convective heating and drying through compensating subsidence directly induced by the updraft flux. Due to uncertainty and computational limit, the micron-to-millimeter-scale microphysical processes in convective updrafts that describe the formation of cloud and precipitating particles are ignored or parameterized very crudely in many CPSs. To improve the representation of microphysical processes of atmospheric convection and its interactions with stratiform clouds and aerosol in the MSKF, the coauthors (Zhang and Song) developed an efficient two-moment microphysics parameterization scheme for convective clouds, which is especially suitable for use in CPSs like the MSKF. Based on the microphysics parameterization for stratiform clouds by Morrison and Gettelman (2008), with modifications to suit convective clouds, the scheme explicitly treats the mass mixing ratio and number concentration of four hydrometeor species (cloud water, cloud ice, rain, and snow) and describes several microphysical processes, including autoconversion, self-collection, collection between hydrometeor species, freezing, cloud ice nucleation and droplet activation by aerosols, and sedimentation. The cloud droplet activation on aerosol is parameterized as a function of the convective updraft vertical velocity, aerosol number concentration, and size distribution, while the ice nucleation parameterization links the ice crystals to aerosol properties, updraft velocity, and air temperature. Thus, this physically-based scheme is suitable for investigating the interaction between convection and aerosol and the indirect aerosol effect on climate and weather. The diagram below provides a bird’s eye view of various microphysical processes included in the scheme. Table S1 shows how the original global model aerosol inputs are mapped into the convective microphysics scheme.

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A bird’s Eye View of Microphysical Processes modeled in the convective microphysics in the MSFK.

For a given convective updraft mass flux \((M_u)\) and detrainment rate \((D_u)\), which are available from conventional convection parameterization, the budget equations for mass mixing ratios, \(q_x\) (in units of kg kg\(^{-1}\)), and number concentrations, \(N_x\) (in units of kg\(^{-1}\)), of cloud water, cloud ice, rain, and snow in saturated updraft of convection can be written as:

\[
\frac{\partial}{\partial z} (M_u q_x) = - D_u q_x + \sigma_u S_x^q
\]

\[
\frac{\partial}{\partial z} (M_u N_x) = - D_u N_x + \sigma_u S_x^N
\]

where \(x\) refers to the subscripts \(c\) for cloud water, \(i\) for cloud ice, \(r\) for rain, and \(s\) for snow. \(\sigma_u = M_u / w_u\) is the fractional area occupied by convective updrafts, \(w_u\) is updraft vertical velocity, and \(S_x^q\) and \(S_x^N\) are the source/sink terms for \(q_x\) and \(N_x\), respectively, per unit cloud area. Note that the detrainment of rain and snow are ignored in practice, but are generalized for conciseness.
The symbolic terms on the right-hand side of above eight equations represent the microphysical processes. These terms include autoconversion of cloud water/ice to rain/snow (subscript $\text{auto}$); accretion of cloud water by rain (subscript $\text{accr}$); accretion of cloud water, cloud ice, and rain by snow (subscript $\text{accs}$); homogeneous (subscript $\text{fhm}$) and heterogeneous (subscript $\text{fhet}$) freezing of cloud water/rain to form ice/snow; the Bergeron-Findeisen process (subscript $\text{Berg}$); fallout of rain and snow (subscript $\text{fallout}$); condensation/deposition (subscript $\text{cond}$); self-collection of rain drops (subscript $\text{aggr}$); self-aggregation of snow (subscript $\text{aggs}$); and activation of cloud condensation nuclei or ice nucleation (subscript $\text{nuc}$).

The gamma distribution has been found to describe the observed drop size distribution of individual convective cells well (e.g., Brandes et al., 2003). Thus, the hydrometeor size distributions are assumed to be represented by gamma functions in this study, as in Morrison and Gettelman (2008), with the spectral slope and intercept parameters $\lambda_x$ and $N_{0x}$ derived from $N_x$, $q_x$, and specified spectra shape parameter $\mu$, as seen below:

$$
\lambda_x = \left[ \frac{\pi \rho N_x \Gamma(\mu + 4)}{6q_x \Gamma(\mu + 1)} \right]^{(1/3)}
$$
where $\mu = 1/(0.0005714 N_c + 0.2714)^2 - 1$ (Martin et al., 1994) for cloud droplet with $N_c$ in units of cm$^{-3}$ and $\mu = 0$ for cloud ice, rain, and snow. Here subscript $x = c,i,r,s$ for cloud water, cloud ice, rain, and snow respectively. $\Gamma$ is the gamma function. Following Morrison et al. (2005), upper and lower bounds for $\lambda_x$ are specified so that the mean hydrometer diameter cannot be larger than 50 $\mu$m or smaller than 2 $\mu$m for cloud droplets, 400 or 10 $\mu$m for cloud ice, 500 or 20 $\mu$m for rain, and 2000 or 10 $\mu$m for snow. The parameter $\lambda_x$ is prevented from exceeding these bounds by adjusting the number concentration in first equation shown above.

The conventional convection parameterization provides net condensation and deposition in convective updrafts by assuming that the cloud air is saturated with respect to water or ice. Thus, here we only parameterize the microphysical processes involving hydrometer transformation from one category to another.

(d1) Autoconversion of cloud water to rain: Autoconversion of cloud water to rain is given by (Khairoutdinov and Kogan, 2000):

$$ P_{\text{auto}}^{q_c} = 1350 q_c^{2.47} N_c^{-1.79}, $$

$$ P_{\text{auto}}^{N_c} = \frac{P_{\text{auto}}^{q_c}}{4\pi \rho_w r_0^3}, $$

where $r_0 = 25 \mu$m is a threshold radius, $\rho_w = 1000kgm^{-3}$ is the liquid water density, and $N_c$ in the first equation above is in cm$^{-3}$.

(d2) Accretion of cloud water by rain: Accretion of cloud water by rain follows Khairoutdinov and Kogan (2000):

$$ P_{\text{accr}}^{q_c} = 67 (q_c q_r)^{1.15}, $$

$$ P_{\text{accr}}^{N_c} = \frac{P_{\text{accr}}^{q_c}}{q_c / N_c}. $$
(d3) Self-collection of rain: Self-collection of rain does not change the rain mass, but it changes the raindrop numbers. Its parameterization follows Beheng (1994):

\[ P_{\text{agg}}^N = -8.0 \times 10^3 N_r q_r \]

where \( q_r \) is in g cm\(^{-3}\) and \( N_r \) in cm\(^{-3}\).

(d4) Autoconversion of cloud ice to snow: Similar to Ferrier (1994), the potential autoconversion of cloud ice to snow is calculated by integration of the cloud ice size distribution greater than the specified threshold size. The estimated mixing ratio and number concentrations are then converted into snow over a specified time scale. The changes in \( q_i \) and \( N_i \) due to autoconversion are given by:

\[ P_{\text{auto}}^{q_i} = -\frac{\pi \rho_i N_{0i}}{6 \tau_{\text{auto}}} \left[ D_{ci}^3 \frac{3 D_{ci}^2}{\lambda_i^2} + 6 \frac{D_{ci}}{\lambda_i^3} + 6 \right] e^{-\lambda_i D_{ci}} \]

\[ P_{\text{auto}}^{N_i} = -\frac{N_{0i}}{\lambda_i \tau_{\text{auto}}} e^{-\lambda_i D_{ci}} \]

where \( D_{ci} = 200 \mu m \) is the threshold size separating cloud ice from snow, \( \rho_i = 500 kg m^{-3} \) is the bulk density of cloud ice, and \( \tau_{\text{auto}} = 3 \) min is the autoconversion time scale.

(d5) Homogeneous and heterogeneous freezing: When temperature is below -40ºC, it is assumed that the cloud liquid water and rainwater are instantly converted to cloud ice and snow by homogeneous freezing. At warmer temperatures, freezing of droplets occurs only through heterogeneous freezing on ice nuclei. Heterogeneous freezing can occur in four different modes. In the deposition mode water vapor deposits on ice nuclei without an intermediate liquid stage. For condensation freezing water vapor condenses on a particle first and then freezes. Immersion freezing occurs when ice nucleates immersed in a supercooled liquid droplet, while contact freezing occurs when dry ice nuclei collide with a supercooled liquid droplet. Immersion mode and contact freezing are thought to be most important in mixed phase clouds (Lohmann and Diehl, 2006).

The immersion freezing is parameterized after Diehl and Wurzler (2004), Lohmann and Diehl (2006), and Hoose et al. (2008):

\[ P_{\text{fusi,imm}}^N = -N_{a,imm} \exp(273.15 - T) \frac{dT}{dt} \frac{q_c}{\rho_v}, \]
where the number of immersion ice nuclei, $N_{a,imm}$, is obtained from the normalized aerosol number concentration weighted by ice nucleating efficiency $B_j$ in the immersion freezing mode.

$$N_{a,imm} = \sum_{j=BC,dust} B_j N_{aer,j} / N_{aer,tot}$$

Here, we consider only black carbon (BC) and dust as efficient ice nuclei in the immersion mode. The freezing efficiencies $B_j$ (black carbon: 0.00291 m$^{-3}$; dust: 32.3 m$^{-3}$) are taken from a compilation of laboratory data by Diehl and Wurzler (2004). The cooling rate due to moist-adiabatic ascent in convection is given by:

$$\frac{dT}{dt} = -g \frac{w_u}{c_p + L \left( \frac{dq}{dT} \right)}$$

This parameterization considers different freezing characteristics of various aerosol compositions and produces more realistic freezing rates (Diehl and Wurzler, 2004) compared to the Bigg equation (Bigg, 1953), which is often used in models to describe immersion freezing.

The parameterization of contact freezing follows Liu et al. (2007):

$$P_{fhet,cnt}^{N_c} = 4 \pi v \rho N_{a,cnt} D_{cnt} / \rho$$

$$P_{fhet,cnt}^{q_c} = \frac{4}{3} \pi v^3 \rho \rho_{fhet,cnt} P_{fhet,cnt}^{N_c}$$

where $v$ is the volume mean droplet radius, and $D_{cnt}$ is the Brownian aerosol diffusivity and is given by:

$$D_{cnt} = \frac{k T C_c}{6 \pi \nu_{cnt}}$$
where \( k \) is the Boltzmann constant, \( \nu \) is the viscosity of air, \( r_{\text{cut}} \) is the aerosol number mean radius, and \( C_c \) is the Cunningham correction factor. \( N_{a,\text{cnt}} \) is the number concentration of contact nuclei, which is parameterized as:

\[
N_{a,\text{cnt}} = N_{a0}(270.16 - T)^{1.3}
\]

with \( N_{a0} \) the number of active ice nuclei. Following Liu et al. (2007), we assume that all contact ice nuclei are mineral dust. In addition, secondary ice production between -3° and -8°C (the Hallett–Mossop process) is also included based on Cotton et al. (1986). The heterogeneous freezing of rain to snow is parameterized following Bigg (1953) and Morrison and Gettelman (2008):

\[
P_{\text{fhet}}^q = 20\pi^2 BN_r\rho_r \exp[A(T_0 - T)]\lambda_r^6
\]

\[
P_{\text{fhet}}^N = \pi BN_r \exp[A(T_0 - T)]\lambda_r^{-3}
\]

where \( T_0 = 273.15K \), \( B = 100m^{-3}s^{-1} \), and \( A = 0.66K^{-1} \).

(d6) Accretion of cloud droplets by snow: Accretion of cloud droplets by snow follows Thompson et al. (2004):

\[
P_{\text{accr}}^q = \frac{\pi a_s q_s \rho E_{cs} N_{0s} \Gamma(b_s + 3)}{4 \lambda_s^{b_s+3}}
\]

\[
P_{\text{accr}}^N = \frac{\pi a_s N_c \rho E_{cs} N_{0s} \Gamma(b_s + 3)}{4 \lambda_s^{b_s+3}}
\]

where \( a_s = 11.72 \, m (1 - b_s) \, s^{-1} \) and \( b_s = 0.41 \) are constants; the collection efficiency for droplet–snow collisions, \( E_{cs} \), is a function of the Stokes number, which depends on the mean radii of the cloud droplets and snow and is calculated the same way as in Morrison and Gettelman (2008) for large-scale clouds. \( N_{0s} \) is the intercept parameter for snow size distribution and is given by Eq. E3.

(d7) Accretion of cloud ice by snow: Accretion of cloud ice by snow is given by:
where $E_{rs} = 0.1$ is the collection efficiency between snow and cloud ice.

**(d8) Accretion of rain by snow:** Accretion of rain by snow in subfreezing conditions is given by Ikawa and Saito (1990):

\[
P_{\text{accs}}^{q_r} = \frac{\pi a_i q_r \rho E_{rs} N_{0r}}{4} \frac{\Gamma(b_s + 3)}{\lambda_{s} b_s + 3}
\]

\[
P_{\text{accs}}^{N_i} = \frac{\pi a_i N_i \rho E_{rs} N_{0s}}{4} \frac{\Gamma(b_s + 3)}{\lambda_{s} b_s + 3}
\]

where $E_{rs} = 1$ is the collection efficiency between snow and rain water, $\gamma = 0.08$ , $\alpha = 1.2$ , and $\beta = 0.95$ for the first equation, and $\alpha = 1.7$ and $\beta = 0.3$ for the second equation.

**(d9) Self-aggregation of snow:** The decrease in $N_s$ due to aggregation (Reisner et al., 1998) is parameterized by:

\[
P_{\text{agg}}^{N_s} = \frac{-I(b_s) a_s E_{ss}}{4 \times 720 \pi} \frac{1 - b_s}{3} \rho \frac{2 + b_s}{3} \frac{2 + b_s}{3} \frac{2 + b_s}{3} \frac{4 - b_s}{3} (N_s \rho)^{\frac{4 - b_s}{3}}
\]

where $E_{ss} = 0.1$ is the collection efficiency among snow particles, and $I(b_s) = 1108$ for $b_s = 0.41$.

**(d10) Ice nucleation:** At temperatures below $-35$ °C the homogeneous ice nucleation and heterogeneous immersion nucleation are parameterized following Liu and Penner (2005) and Liu et al. (2007). This parameterization is able to represent the combination of homogeneous and heterogeneous nucleation pathways and to link the ice crystals to aerosol properties, updraft
velocity, and air temperature. The potential ice crystal number density \( N_{i,\text{hom}} \) (cm\(^{-3}\)) from homogeneous nucleation, as a function of \( T \), \( w_u \), and sulfate aerosol number concentration \( N_a \) (cm\(^{-3}\)), is given by:

\[
N_{i,\text{hom}} = \begin{cases} 
\min \{ \exp(a_2 + b_2 T + c_2 \ln w_u) N_a^{a_1 + b_1 T + c_1 \ln w_u}, N_a \}, & T \geq 6.07 \ln w_u - 55 \\
\min \{ \exp[a_2 + (b_2 + b_3 \ln w_u) T + c_2 \ln w_u] N_a^{a_1 + b_1 T + c_1 \ln w_u}, N_a \}, & T < 6.07 \ln w_u - 55
\end{cases}
\]

Here the values of coefficients are the same to those used in Liu et al. (2007). In the present study we assume only dust and hydrophilic black carbon particles to be potential immersion nuclei. The potential number of ice crystals formed from immersion nucleation \( N_{i,\text{het}} \) (cm\(^{-3}\)) is given by:

\[
N_{i,\text{het}} = \min \{ \exp[(a_2 \ln w_u + a_{22}) + (a_{11} \ln w_u + a_{12}) T] N_h^{(b_{21} \ln w_u + b_{22}) + (b_{11} \ln w_u + b_{12}) T}, N_h \}
\]

where \( N_h \) (cm\(^{-3}\)) is dust and hydrophilic black carbon number concentration and the values of coefficients are also the same to those used in Liu et al. (2007). In addition, the potential ice crystal number density \( N_{i,\text{dep}} \) (l\(^{-1}\)) from the deposition/condensation nucleation on mineral dust between -40\(^\circ\) and 0\(^\circ\)C is represented by the Meyers et al. (1992) formulation:

\[
N_{i,\text{dep}} = \exp \{ a + b[100(RH_i - 1)] \}
\]

with \( a=-0.639 \) and \( b=0.1296 \). The ice nucleation rate due to homogeneous, heterogeneous immersion, and deposition/condensation ice nucleation are given by:

\[
P_{\text{nuc, hom}} = \max \{ (N_{i,\text{hom}} - N_i) / \tau, \ 0 \}
\]

\[
P_{\text{nuc, het}} = \max \{ (N_{i,\text{het}} - N_i) / \tau, \ 0 \}
\]

\[
P_{\text{nuc, dep}} = \max \{ (N_{i,\text{dep}} - N_i) / \tau, \ 0 \}
\]

where \( N_i \) is the existing ice number concentration, and the time scale of ice nucleation is assumed to be 30 min.

(d11) Bergeron–Findeisen process: In temperature ranges where both liquid and ice water coexist, since ice saturation vapor pressure is lower than water saturation vapor pressure, ice
crystals may grow at the expense of liquid water through the Bergeron–Findeisen process. Following Lohmann et al. (2007), the Bergeron–Findeisen process is parameterized as a threshold process. We assume that when ice water exceeds a threshold value of 0.5 mg kg⁻¹, the existing liquid water evaporates within a single time step, and the water mass is deposited onto the existing ice crystals.

(d12) Fallout of rain and snow: Similar to Kuo and Raymond (1980), the fallout of rain and snow is parameterized as:

\[ P_{\text{fallout}}^q = \frac{V_q}{\Delta z} q_s \]

\[ P_{\text{fallout}}^N = \frac{V_N}{\Delta z} N_s \]

Here \( V_q \) and \( V_N \) are the fall speeds at which rain/snow mass and the number of rain/snow particles leave the model layer, respectively. The mass- and number-weighted terminal fall speeds for all precipitation species are obtained by integration over the particle size distributions with appropriate weight of mixing ratio or number concentration:

\[ V_N = \left( \frac{\rho_{a0}}{\rho} \right)^{0.54} \frac{a\Gamma(1+b)}{\lambda_s^b} \]

\[ V_q = \left( \frac{\rho_{a0}}{\rho} \right)^{0.54} \frac{a\Gamma(4+b)}{6\lambda_s^b} \]

where \( \rho_{a0} \) is the reference air density at 850mb, \( a=841.997m^{1-b} s^{-1} \) (\( a=11.72m^{1-b} s^{-1} \)) and \( b=0.8 \) (\( b=0.41 \)) are empirical coefficients in the diameter–fall speed relationship \( V = aD^b \) for rain (snow), where \( V \) is the terminal fall speed for a particle with diameter \( D \). The air density correction factor is adopted from Heymsfield et al. (2007). The \( V_N \) and \( V_q \) are limited to maximum values of 10 m s⁻¹ for rain and 3.6 m s⁻¹ for snow. Once the precipitation particles fall out of the updrafts, a Sundqvist (1988) style evaporation of the convective precipitation is employed in the subsaturated model layer.

(d13) Activation of cloud droplets on aerosol: The parameterization of cloud droplet nucleation for multiple aerosol types with different size distribution by Abdul-Razzak and Ghan (2000) is
adopted in this study. This parameterization can account for the competition between different aerosol types as cloud condensation nuclei, in which the number concentration of droplets is parameterized as a function of updraft vertical velocity, temperature, aerosol number concentration, and size distribution. The convective updraft vertical velocity derived from section 2.2 is applied in the activation of cloud droplets. Following Morrison and Gettelman (2008), the existing $N_c$ is used as a proxy for the number of aerosols previously activated, and an activation time scale of 30 min is assumed.

Stabilization Capacity

In many CPSs the convective available potential energy (CAPE) is removed by about 90% of the available CAPE such that stability is restored to an atmospheric grid column by removing the convective instability. This kind of ad hoc assumption may be valid and working well at coarse resolutions. However, when such CPSs are used in grey zones scale, there is a need to control the ability of a CPS in restoring the stability. Else, the gradual handover of restoring stability to a grid scale cloud microphysical scheme will be hampered across these high-resolution scales. To facilitate this feature in the MSKF, the CAPE removal is scaled as per Eq. E1. This process of controlling the stabilization capacity (SC) by the MSKF is represented as:

$$A_e^r = (1 - \gamma \beta) A_e$$

where $A_e^r$ is entrained CAPE to be removed from a convective cloud, $\gamma = 0.1$ which makes the code remove 90% of the CAPE, and $\beta$ is the factor as shown in Eq. E1. Thus, the above equation facilitates the gradual reduction in the CAPE removal in the MSKF across the grey zone scales.

Dynamic Lateral Entrainment

A key convective cloud process is the interactions between convection and its environment through entrainment and detrainment. These processes are quite complex and are of vital importance in regional and global models (e.g., Tokioka et al., 1988; Kain and Fritsch, 1990; Kang et al., 2009). In many global models (e.g., Neale et al., 2010), the entrainment rate is specified and is a parameter often adjusted to improve results; however, there are very few regional and global models in which the entrainment rate is empirically estimated (e.g., Kain, 2004; Chikira and Sugiyama, 2010; Del Genio et al., 2012). But for grey zone scale simulations, assumptions made in an entrainment formulation need to be revisited.

From Kain (2004) the equation of the minimum entrainment rate for convective plumes is given by:

$$\Delta M_e = M_b \frac{C}{R} \Delta P$$
where $\Delta M_e$ is the mixing rate (kg s$^{-1}$), $M_b$ is the updraft mass rate at cloud base (kg s$^{-1}$), $C=0.03$ is a constant (m Pa$^{-1}$) which controls the overall magnitude of the entrainment rate for convective plumes, $R$ is the radius of cloud base and dependent on the magnitude of vertical velocity at the lifting condensation level (LCL) (m), and $\Delta P$ is the pressure depth of a model layer (Pa). The magnitude of the constant $C$ used in the above equation is same as that of the non-dimensional Tokioka parameter, $\alpha = 0.03$, (Tokioka, 1988) used in global climate studies (e.g., Kang et al., 2009; Kim et al., 2011; Lin et al., 2013) for entrainment rate estimation. These global studies showed that the hyper-activity of a subgrid scale convection scheme can be largely modulated by tuning the Tokioka parameter, which allowed grid scale processes to perform the needed moisture conditioning of large-scale atmosphere. These studies also showed that the subgrid scale precipitation decreases as the Tokioka parameter increases, resulting in an increase of grid scale precipitation improving climate simulations. Dependence of the entrainment on horizontal grid resolution for radiatively driven shallow (stratocumulus) clouds was studied by Stevens and Bretherton (1999) using a large-eddy simulation model. In their study it was found that when the horizontal spacing is coarsened, the entrainment rate decreased without any noticeable changes in overall structures of the subcloud layer and cloud layer. The role of entrainment for continental deep convective clouds was extensively studied by Del Genio and Wu (2010). One of their findings was that at higher spatial resolutions their inferred entrainment rate was also higher because turbulence was more resolved. They also used the WRF model at different grid resolutions and found the inferred entrainment rate at 125 m grid resolution to be stronger than that inferred at 600 m grid spacing. Entrainment in deep convective clouds was also studied by Romps and Kuang (2010) using a LES model. It was shown that the purity of convection decreases with finer grids (ranging from 3200 to 100 m spacings) suggesting increased entrainment with increased grid spacing. Finally, in a recent cloud resolving modeling study, Bryan and Morrison (2012) concluded that changes in the simulated squall line intensity differences between two model grid resolutions (1 and 0.25 km) was primarily attributed to the increased entrainment. Thus, all these studies clearly highlighted the dependency of entrainment on the horizontal grid resolution (i.e., entrainment increases as grid resolution increases). De Rooy et al. (2013) provides a detailed review of entrainment in cumulus convection and highlights the study of Houghton and Cramer (1951) that entrainment needs to be partitioned into two parts: (1) entrainment due to large-scale processes; and (2) entrainment due to turbulence at cloud edges. Since the first type of entrainment is being represented by Eq. 8, we have included the second type of entrainment through the usage of the Tokioka parameter.

We considered all these findings when reformulating the entrainment rate to make it more adaptable to grey zone scales and to work seamlessly across spatial scales. We introduce this feature through the usage of the dynamic Tokioka parameter that increases as model resolution increases. The resolution dependent Tokioka parameter helps to represent grid spacing effects on convective cloud-entrainment interactions similar to that documented in the literature. Hence, consistent with the above global climate and large-eddy simulation studies, we have introduced a scale dependency for the Tokioka parameter by multiplying it with $\beta$ shown in Eq. 3 and also replaced $R$ with $Z_{LCL}$ (m) – subcloud layer depth – which is the height of the LCL above the ground, again consistent with the studies mentioned earlier. The main advantage of using $Z_{LCL}$ instead of $R$ is that at higher resolutions, $R$ generally approaches an upper limit of 2 km used in the KF scheme; thus, it is not consistent with the assumption that subgrid scale cloud fraction covers only a small area of a grid cell (e.g., Arakawa and Wu, 2013). In such situations the
diameter of a KF cloud will become 4 km, and therefore at 3 km grid spacing used in this study, usage of $R$ is inappropriate as the assumed subgrid cloud diameter exceeds the grid size.

Then, the new minimum entrainment equation can be written as:

$$\Delta M_e = M_b \frac{\alpha_\beta}{z_{LCL}} \Delta P$$

The above new entrainment equation attempts to include both the types of entrainments consistent with the descriptions of de Rooy et al. (2013) and is a scale-aware formulation. For further details and evaluation of the performance of the above equation, readers are referred to Zheng et al. (2016).

Enhancement of Grid Scale Vertical Velocity Using Subgrid Scale Updraft Mass Fluxes

Many studies (e.g., Han and Pan, 2011; Richter and Rasch, 2008; Mallard et al., 2013) cite the need for inclusion of convective momentum transport into the KF scheme for proper simulation of hurricanes. But, for high-resolution convective precipitation forecasts it is not clear whether subgrid scale updraft mass flux plays an important role on grid scale momentum, mass, and energy transport. To address an aspect of this issue, we considered impacts of subgrid scale updraft mass fluxes on grid scale vertical velocity using a simple linear methodology. One potential benefit is that it can help reduce model spin up time over convectively active regions through increasing the grid scale vertical velocity. The proposed simple linear mixing methodology for enhancing grid scale vertical velocity is expressed as:

$$W_{up} = \frac{M_{up}}{\rho} = \frac{M}{\rho}$$

$$W_n = W_g + W_{up}$$

where $W_{up}$ is the effective vertical velocity of subgrid scale updraft (m s$^{-1}$), $M_{up}$ is the subgrid scale updraft mass flux (kg m$^{-2}$ s$^{-1}$), $\rho$ is the convective plume density (kg m$^{-3}$), $M$ is the updraft mass rate (kg s$^{-1}$), $W_n$ is the reformulated grid scale vertical velocity (m s$^{-1}$), and $W_g$ is the grid scale vertical velocity (m s$^{-1}$). For further details and evaluation of the performance of the above equation, readers are referred to Zheng et al. (2016).
Flow Diagram of Precipitation Estimation in MSKF and MDM schemes

The below given flow chart shows how total surface precipitation is estimated when using the MSKF and MDM schemes in the WRF model.
D = Detrainment of condensates to grid scale scheme
Table S1 Mapping of Aerosol Species from CESM-NCSU to WRF-ACI

<table>
<thead>
<tr>
<th>SZ11 Bulk Aerosol Name</th>
<th>CESM-NCSU Aerosol Mode</th>
<th>CESM-NCSU Species</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sulfate</td>
<td>Aiken and Accumulation</td>
<td>Sulfate, Nitrate, and Ammonium</td>
</tr>
<tr>
<td>Sea-salt</td>
<td>Fine Sea-salt and Coarse Sea-Salt</td>
<td>Sulfate, Nitrate, Ammonium, Sodium, and Chloride</td>
</tr>
<tr>
<td>Dust 1</td>
<td>50% of Fine Dust</td>
<td>Sulfate, Nitrate, Ammonium, and Mineral Dust</td>
</tr>
<tr>
<td>Dust 2</td>
<td>50% of Fine Dust</td>
<td>Sulfate, Nitrate, Ammonium, and Mineral Dust</td>
</tr>
<tr>
<td>Dust 3</td>
<td>50% of Coarse Dust</td>
<td>Sulfate, Nitrate, Ammonium, and Mineral Dust</td>
</tr>
<tr>
<td>Dust 4</td>
<td>50% of Coarse Dust</td>
<td>Sulfate, Nitrate, Ammonium, and Mineral Dust</td>
</tr>
<tr>
<td>Hydrophilic Black Carbon</td>
<td>40% of Accumulation</td>
<td>Black Carbon</td>
</tr>
<tr>
<td>Hydrophobic Black Carbon</td>
<td>Primary Carbon and 60% of Accumulation</td>
<td>Black Carbon</td>
</tr>
<tr>
<td>Hydrophilic Organic Carbon</td>
<td>Aiken and Accumulation</td>
<td>Secondary and Semi-Volatile Organic Aerosol</td>
</tr>
<tr>
<td>Hydrophobic Organic Carbon</td>
<td>Primary Carbon and Accumulation Mode</td>
<td>Primary Organic Aerosol</td>
</tr>
</tbody>
</table>
Figure S1: Atmospheric precipitable water vapor from both the BASE and WACI simulations.
Figure S2: Differences in cloud droplet number concentration and ice number concentration between the WACI and LAERO simulations.
Figure S3: Difference in source, sink, and tendency terms for subgrid scale and grid scale cloud liquid water between the WACI and LAERO simulations.
Figure S4: Difference in source, sink, and tendency terms for subgrid scale and grid scale cloud ice water between the WACI and LAERO simulations
Figure S5: Difference in source, sink, and tendency terms for subgrid scale and grid scale rain water between the WACI and LAERO simulations.
Figure S6: Difference in source, sink, and tendency terms for grid scale snow and graupel water between the WACI and LAERO simulations.
Figure S7: Difference in source, sink, and tendency terms for subgrid scale snow water between the WACI and LAERO simulation
Figure S8: Difference in subgrid scale and grid scale net precipitation production rate between the WACI and LAERO simulations
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