Supplemental Material for

Summertime Planetary Wave Resonance in the Northern and Southern Hemispheres

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Effective Forcing Amplitude

In case a waveguide is present for a free synoptic wave with wavenumber $k$ (i.e. criterion $i$ is fulfilled), and under the assumption that the frictional force is acting mainly in the planetary boundary layer (PBL), then Eq. [S14] from (Petoukhov et al. 2013) for the wave amplitude is valid:

$$A_m = \frac{A_{\text{eff}}}{\sqrt{[\frac{(k/a)^2 - (m/a)^2}{2} + \frac{L/a^2 + R^2/L}{2} (m/a)^2]}}$$  \[6\]

Here, $m$ is the wavenumber of the forced wave, $R = \kappa R_0$ (with $R_0 = 0.135$) is the Rossby number for eddies dominantly contributing to the efficient atmospheric “eddy viscosity” (friction force in the PBL) and $L = \kappa L_0$ (with $L_0 = 6 \cdot 10^5$ m as the characteristic Rossby radius of the above-mentioned eddies). The amplitude of the effective forcing $A_{\text{eff}}$ can be calculated by applying a zonal FFT on the area-weighted meridional average of the effective wave-forcing $F_{\text{eff}}$:

$$A_{\text{eff}} = \text{FFT}(F_{\text{eff}}),$$  \[7\]

with $F_{\text{eff}}$ at 300 mb calculated by employing Eq. [S1c] from (Petoukhov et al. 2013):

$$F_{\text{eff}} = \frac{2 \Omega \sin(\phi) \cos(\phi)^2}{a T_c} \frac{\partial \hat{T}}{\partial \lambda} + \frac{2 \Omega \sin(\phi) \cos(\phi)^2}{a H} K \frac{\partial h_{\text{or}}}{\partial \lambda},$$  \[5\]

where $\lambda$ is longitude, $T_c = 200$ K is a constant reference temperature at the equivalent barotropic level (EBL), $\hat{T}$ is the 15-days running mean azonal temperature at 300 mb, $H = 12000$ m is the characteristic scale of the troposphere height, $\kappa$ is the characteristic value of the ratio of the zonally averaged zonal wind at the mean orographic height $h_{\text{or}}$ of the coarse resolution orography and the zonally averaged zonal wind $U$ at 300 mb ($\kappa_{NH} \approx 0.4, \kappa_{SH} \approx 0.7$).

Apart from $k$ being close to $m$, for efficient amplification to take place, $A_{\text{eff}}$ has to be of sufficient magnitude. Therefore $A_{\text{eff}}$ is determined for wavenumbers $1 - 15$, and a threshold quantile $q_k$ is estimated. In our analysis we set this threshold to the median forcing, implying that the forcing for the trapped wave has to be among the strongest 60%.

Amplitude test

As a test, we check whether the calculated wave amplitude (Eq. [6]) is close to those observed during periods when resonance conditions are fulfilled ($i$. + $ii.$). We calculate $A_m$ for $k = m \pm 0.2$ giving an estimated range of wave amplitudes expected from resonance. When the observed wave amplitude falls within this range, the Amplitude test (AT) is passed. For $k = m \pm 0.2$, Eq. [6] estimates typically rather high amplitudes $A_m$. Eq. [6] shows that the zonal wavenumber $k$ needs to be close to the zonal wavenumber $m$ of the forced wave. In that case the first term in the denominator is close to zero resulting in large values of $A_m$, i.e. strong amplification. Petoukhov et al. (2016) approximated the maximum difference $|k - m|$ to be between $0.25 - 0.30$ depending on wavenumber. This was determined by finding the maximum difference $|k - m|$ which leads to
an amplitude $A_m$ of $1.5 \sigma$ threshold above the 1979-2015 climatology, as calculated with Eq. [6] and
with $A_{\text{eff}}$ set to the maximum observed value during 1979-2015. In our implementation a
conservative approximation is applied, setting the maximum difference of $k - m = \pm 0.2$. This
leads to a range of considered values of $k$ as follows: $k_6 = 5.8 - 6.2$, $k_7 = 6.8 - 7.2$, $k_8 = 7.8 -
8.2$.

The AT thus tests whether the high amplitudes expected from resonance are actually observed. Due
to the limitations of the approach as for example zonal mean field analyses or the use of linearized
equations (see discussion) there can be cases a waveguide is detected but still the associated wave
is not efficiently trapped.

We apply the AT to single clusters of consecutive days fulfilling the conditions i. and ii. only, and not
to each individual day. The reason is that Eq. [6] provides a stationary solution for the eventual
possible wave amplitude. However, it takes several days for resonance to amplify waves and thus
the first days fulfilling i. and ii. don’t necessarily show exceptional amplitudes. Also nonlinear
processes such as Rossby wave breaking are not captured by Eq. [6].

QRA-clusters are defined as sequences of consecutive time steps meeting i. and ii. for at least one $k$.

For a cluster of QRA days, the AT needs to be fulfilled for at least 25% of days which was found to
give reasonable results.

Refractive index

The squared meridional wavenumber (Eq. [1]) in our analysis serves as an index of refraction in the
case of circumglobal planetary waves with high zonal wavenumber ($k \geq 4$) in the mid-latitudes.

In the following we will present how barotropic index of refraction (as shown in (Branstator 1983)
is related to the squared meridional wavenumber derived by Petoukhov et al. (2013).

An Expression for the barotropic index of refraction for planetary waves was derived by Branstator
(1983) (see his Eq. [4b]):

$$\rho^2 = \frac{\cos \varphi}{\bar{\omega} - i \alpha k^{-1}} \frac{\partial \bar{q}}{\partial \varphi} - k^2, \quad [S1]$$

where $i$ is imaginary unit, $\varphi$ the latitude and -for circumglobal planetary waves- $\bar{q}$ is the zonally
averaged potential vorticity (see his Eq. [3]) described as

$$\bar{q} = \frac{1}{a^2 \cos \varphi} \frac{\partial}{\partial \varphi} \left( \cos \varphi \frac{\partial \bar{\psi}}{\partial \varphi} \right) + f, \quad [S2]$$

with $a$ the Earth’s radius, $f$ the Coriolis parameter, $\bar{\psi}$ the zonally averaged stream function, and the
zonally averaged angular velocity $\bar{\omega}$ given by

$$\bar{\omega} = -\frac{1}{a^2 \cos \varphi} \frac{\partial}{\partial \varphi} \left( \cos \varphi \frac{\partial \bar{\psi}}{\partial \varphi} \right) = \frac{1}{a \cos \varphi} \bar{u}, \quad [S3]$$

with $\bar{u}$ the zonally averaged zonal flow, while $\alpha$ and $k$ are the “eddy viscosity” coefficient and the
zonal wavenumber, respectively. Substitution of [S2] and [S3] into [S1] yields:
\[ \rho^2 = \frac{\cos \varphi}{a \cos \varphi} \left( \frac{1}{i\kappa^{-1} a} \left( \frac{\sin \varphi \partial \bar{u}}{\cos \varphi \partial \varphi} + \frac{\bar{u}}{\cos^2 \varphi} \frac{\partial^2 \bar{u}}{\partial \varphi^2} + 2\Omega a \cos \varphi \right) - k^2 \right). \]  

[S4]

Eq. [S4] gives a general expression for the square of the refractive index. With characteristic values of \( \bar{u} \approx 10 \text{ ms}^{-1} \) and \( \alpha \approx 10^{-6} \text{ s}^{-1} \) ((Branstator 1983; Petoukhov et al. 2013) in the midlatitudes \( \varphi \approx 45^\circ \)), the first term in the denominator of the first fraction on the right hand side of [S4] \( \left( \frac{\bar{u}}{a \cos \varphi} \right) \) is at least an order of magnitude greater than the second \( (i\alpha^{-1} \kappa) \), for \( k \geq 4 \). Thus, as an approximation for waves of higher zonal wavenumbers \( (k \geq 4) \) in the midlatitudes [S4] can be written as:

\[ \rho^2 \approx \cos^2 \varphi \left( \frac{\sin \varphi \partial \bar{u}}{\cos \varphi \partial \varphi} + \frac{\bar{u}}{\cos^2 \varphi} \frac{\partial^2 \bar{u}}{\partial \varphi^2} + 2\Omega a \cos \varphi \right) - k^2. \]  

[S4a]

After division by \( a^2 \) we obtain an expression that equals the meridional wavenumber \( l^2 \) as derived by Petoukhov et al. (2013):

\[ \frac{\rho^2}{a^2} \approx \frac{\cos^2 \varphi}{\bar{u}} \left( \frac{\sin \varphi \partial \bar{u}}{\cos \varphi \partial \varphi} + \frac{\bar{u}}{\cos^2 \varphi} \frac{\partial^2 \bar{u}}{\partial \varphi^2} + 2\Omega a \cos \varphi \right) - \frac{k^2}{a^2} = l^2 \]  

[S5]

### Estimation of the Rossby number \( R_0 \) and the Rossby Radius \( L_0 \).

The term \( \frac{L}{a^2} + \frac{R^2}{L} \) (with Rossby number \( R = \kappa R_0 \) with \( R_0 = 0.135 \) and Rossby Radius \( L = \kappa L_0 \) with \( L_0 = 6 \cdot 10^5 \text{ m} \)) in the denominator of the fraction in the right hand side of Eq. [6] in our paper describes the \( \frac{1}{U} \left( \frac{k_h}{a^2} + \frac{k_z}{H^2} \right) \) quantity, entering the friction term in the right hand side of the applied barotropic vorticity equation, where \( U \) is the zonal mean zonal flow at the EBL, \( k_h \) and \( k_z \) the representative horizontal and vertical coefficients of eddy viscosity, while \( a \) is the earth’s radius and \( H \) is the atmospheric density vertical scale (see Eqs. [S1], [S1a], [S1c] and [S7] in Petoukhov et al. (2013)). This way, \( L \) and \( R \) in the above term are described by \( L = \kappa L_0 \) and \( R = \kappa R_0 \), where \( \kappa \) is the ratio of the zonally averaged zonal wind at the mean orographic height and \( U \) at the EBL, while \( L_0 = 6 \cdot 10^5 \text{ m} \) and \( R_0 = 0.135 \) are the characteristic Rossby horizontal length scale and Rossby number for the eddies dominantly contributing to the atmospheric “eddy” friction (AEF) in the considered barotropic equation.

To obtain the above estimations of \( L_0 \) and \( R_0 \) we apply a scale-magnitude analysis to \( k_h \) and \( k_z \) (Monin 1972):

\[ k_h = \bar{U} \bar{L}, \quad k_z = \bar{W} \bar{H}, \]  

[S6]

where \( \bar{U}, \bar{W} \) and \( \bar{L}(=L_0), \bar{H} \) are the characteristic velocities and linear scales of the above mentioned atmospheric eddies (hereafter AEF eddies), which dominantly contribute to the atmospheric eddy friction are in the considered case (hereafter, AEF eddies). With the use of the continuity equation, \( \bar{W} \) and \( \bar{H} \) can be parameterized as follows (Monin 1972):

\[ \bar{W} \approx \frac{\bar{U}}{L_0} \bar{H}, \quad \bar{H} \approx HR_0, \]  

[S7]
where $\tilde{R}_0 = \frac{\vartheta}{L_0} f$ is the Rossby number for these eddies. The term $\frac{1}{U} \left( \frac{k_h}{a^2} + \frac{k_z}{H^2} \right)$ can be then represented by

$$\frac{1}{U} \left( \frac{k_h}{a^2} + \frac{k_z}{H^2} \right) = \frac{\bar{U}}{U} \left( \frac{L_0}{a^2} + \frac{R_0^2}{a^2} \right) = \frac{\bar{U}L}{a^2} + \frac{\bar{R}_0^2}{\bar{L}_0} = \frac{L}{a^2} + \frac{R^2}{L}, \quad [3]$$

with $\bar{R} = \bar{U}/L = \bar{K}L_0$ and $R = \bar{R}R_0$. We further set $\tilde{U} \approx U_{or}$, thus allowing that AEF eddies are those ones that act in the vicinity of the average height of the coarse resolution orography, $\bar{H}_{or}$, so that $\bar{K} \approx \kappa$. Further, following Helds (1999) scale-magnitude analysis (see Eq.(14) in that paper), we parameterize $\bar{L}$ in the form

$$L_0 = \frac{\bar{U}}{U} \lambda = \kappa \lambda, \quad [S8]$$

where

$$\lambda = \frac{NH}{f} \quad [S9]$$

is the internal Rossby deformation radius (Pedlosky 1979), as the representative horizontal scale, $\bar{L}_b$, for the mid-latitude synoptic scale transients, with $f$ as the Coriolis parameter and $N$ as the Brunt-Väisälä frequency at the EBL

$$N = \left[ \frac{g}{\vartheta_b} (\Gamma_a - \Gamma_b) \right]^{1/2}, \quad [S10]$$

where $g$ is the gravity acceleration, $\vartheta_b$ is the potential temperature at the EBL, while $\Gamma_a$ and $\Gamma_b$ are respectively the adiabatic lapse rate and the characteristic atmospheric lapse rate in the free troposphere. Following Stone (1972) we require that the characteristic value, $\bar{U}_b$, of the horizontal velocity for the synoptic scale transients at the EBL is close to the zonal mean zonal flow $U$ at this level. Substitution of $\Gamma_a \approx 9.8 \cdot 10^{-3}$ k/m, $\Gamma_b \approx 5 \cdot 10^{-3}$ k/m, $g \approx 9.81 \text{m/s}^2$, $\vartheta_b \approx 325 \text{K}$, $U \approx 13.5 \text{m/s}$, $H \approx 8 \cdot 10^3 \text{m}$, $\kappa = 0.4 - 0.7$ and $f \approx 10^{-4}$ s$^{-1}$, where $\vartheta_b$, $U$ and $\kappa$ correspond to the values of these variables averaged over the summer mid-latitudes of both Hemispheres, just yields $R_0 \approx 0.135$ and $L_0 \approx 6 \cdot 10^5$ as employed in Eq. [6] of our paper.
Hemispheric circulation characteristics

As in Fig. 1 but with $U$ and $c$ seasonally resolved. The NH exhibit a strong seasonal variability between JJA and DJF distributions, while in the SH these differences are less pronounced.
**Statistical Significance**

**Fig.S2 | NH**: Probability density distribution of Amplitudes of quasi-stationary waves $|c| \leq 2 \text{ m/s}$ of waves 6 – 8 during QRA periods (red) and non-QRA periods (black). The sample size $N$ is provided in the upper right corner, the p-value from a Kolmogorov-Smirnov test is given in the upper left corner. A p-value of 0.05 served as a criterion for statistical significance. Statistically significant shifts are observed for wave 7 (i-AT) and wave 6 (ii. – AT).
Fig.S3 | SH: Probability density distribution of Amplitudes of quasi-stationary waves $|c| \leq 2 \text{ m/s}$ of waves 4 – 6 during QRA periods (red) and non- QRA –periods (black). The sample size N is provided in the upper right corner, the p-value from a Kolmogorov-Smirnov test is given in the upper left corner. A p-value of 0.05 served as a criterion for statistical significance. Statistically significant shifts are observed for wave 5 (i.- AT.) and wave 6 (ii.-AT). Wave 6 shows statistical significant differences (i-AT), but has a very small sample size (N=5).
Sensitivity Analysis

Latitudinal Range = 60°S – 30°S, $\kappa = 0.4$

Fig. S4
### Fig. S5

**Tab. S1**

<table>
<thead>
<tr>
<th>Condition/Wave</th>
<th>Wave 4 Events/days</th>
<th>Wave 5 Events/days</th>
<th>Total Events/days</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>205 / 2235</td>
<td>147 / 1008</td>
<td>352 / 3243</td>
</tr>
<tr>
<td>i+ii</td>
<td>230 / 1789</td>
<td>159 / 814</td>
<td>389 / 2603</td>
</tr>
<tr>
<td>i+ii+AT</td>
<td>206 / 1715</td>
<td>138 / 744</td>
<td>344 / 2333</td>
</tr>
</tbody>
</table>
Latitudinal Range = 57.5°S – 37.5°S, \( \kappa = 0.7 \)

Fig. S6
### Fig. S7

<table>
<thead>
<tr>
<th>Condition/Wave</th>
<th>Wave 4</th>
<th>Wave 5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>SH i</td>
<td>209 / 2313</td>
<td>151 / 1015</td>
<td>360 / 3328</td>
</tr>
<tr>
<td>i+ii</td>
<td>230 / 1842</td>
<td>144 / 710</td>
<td>374 / 2552</td>
</tr>
<tr>
<td>i+ii+AT</td>
<td>190 / 1699</td>
<td>100 / 555</td>
<td>290 / 2254</td>
</tr>
</tbody>
</table>

**Running mean averaging**

**11-days running mean**
Fig. S8 | NH: Probability density distributions of wave speed vs. wave-amplitude of detected QRA days based on 11-day running means in the NH for waves 6-8 after applying conditions i (top row), i+ii (central row) and the AT i+ii+iii (bottom row). Anomalies are shown in color together with the contours of JJA climatology (black solid curves). The area of stationary waves (|c| = ±2 m/s) is depicted by dashed black vertical lines. Statistical significant differences for those waves are indicated with a red cross in the upper right corner of each subplot. The sample size N for each distribution is given in the top left corner.
Fig. S9 | SH: Probability density distributions of wave speed vs. wave-amplitude of detected QRA days based on 11-day running means in the SH for waves 4-5 after applying conditions i (top row), i+ii (central row) and the AT i+ii+iii (bottom row). Anomalies are shown in color together with the contours of DJF climatology (black solid curves). The area of stationary waves ($|c| = \pm 2\, \text{m/s}$) is depicted by dashed black vertical lines. Statistical significant differences for those waves are indicated with a red cross in the upper right corner of each subplot. The sample size $N$ for each distribution is given in the top left corner.
Fig. S10 | Probability density plots of detected waveguide’s first turning point vs their widths based on 11-day running means for b) NH for waves 6–8 and a) SH for waves 4, 5 (colored contours) and all respective wavenumbers combined (grey shading). The position of waveguides shifts South and decreased widths with increasing wavenumber. b) In the NH waveguides wave waveguides are located at latitudes 35°N – 40°N for waves 7–8, whereas waveguides of waves 6 are spread over latitudes 40°N -50°N. a) In the SH Waveguide positions are distributed over a wider latitudinal range.
Fig. S11| Probability density distributions of waves phase positions for waves 4 – 5 based on 11-day running means (SH, left) and waves 6 – 8 (NH, right) as summer climatology (grey dashed lines), during detected QRA events (colored) and non-QRA days (black). Maxima occurring in the QRA phase distributions are marked with $p_1$ and $p_2$ in the order of magnitude.
**Fig. S12 | NH:** Probability density distributions of wave speed vs. wave-amplitude of detected QRA days based on 19-day running means in the NH for waves 6-8 after applying conditions i (top row), i+ii (central row) and the AT i+ii+iii (bottom row). Anomalies are shown in color together with the contours of JJA climatology (black solid curves). The area of stationary waves ($|c| = \pm 2 \text{ m/s}$) is depicted by dashed black vertical lines. Statistical significant differences for those waves are indicated with a red cross in the upper right corner of each subplot. The sample size $N$ for each distribution is given in the top left corner.
Fig. S13 SH: Probability density distributions of wave speed vs. wave-amplitude of detected QRA days based on 19-day running means in the SH for waves 4-5 after applying conditions i (top row), i+ii (central row) and the AT i+ii+iili (bottom row). Anomalies are shown in color together with the contours of DJF climatology (black solid curves). The area of stationary waves ($|c| = \pm 2 \text{ m/s}$) is depicted by dashed black vertical lines. Statistical significant differences for those waves are indicated with a red cross in the upper right corner of each subplot. The sample size $N$ for each distribution is given in the top left corner.
Fig.S14 | Probability density plots of detected waveguide’s first turning point vs their widths based on 19-day running means for b) NH for waves 6-8 and a) SH for waves 4, 5 (colored contours) and all respective wavenumbers combined (grey shading). The position of waveguides shifts South and decreased widths with increasing wavenumber. b) In the NH waveguides wave waveguides are located at latitudes 35°N – 40°N for waves 7 – 8, whereas waveguides of waves 6 are spread over latitudes 40°N -50°N. a) In the SHWaveguide positions are distributed over a wider latitudinal range.
Fig. S15 | Probability density distributions of waves phase positions based on 19-day running means for waves 4 – 5 (SH, left) and waves 6 – 8 (NH, right) as summer climatology (grey dashed lines), during detected QRA events (colored) and non-QRA days (black). Maxima occurring in the QRA phase distributions are marked with $p_1$ and $p_2$ in the order of magnitude.
Fig. S16: Composite Meridional wind-fields during non QRA days (a,c) and summer climatology (b,d) in NH and SH
Fig. S17 | a) SH 15 day running mean of $U$ as DJF climatology (black) and variability is shown as a probability density (red shadings). Here the $\pm 1\sigma$ mark is depicted with black dashed lines respectively. b,g) Density of $l^2$, c,h) first term determining the general height and d,i) second term determining the waveguide positions during QRA days. The mean is depicted in black while $\pm 1$ std. is shown in dashed black lines.e,j) Probability density of the average central latitude of waveguide vs waveguide width.

In all panels the mean positions of the equatorward TP for free waves (SH: $\geq 4$, NH: $\geq 6$) is indicated by a vertical black line. The mean value of $f$ at these latitudes is given in red at the respective subplots h,c). Taking into account the uncertainty of d) we obtain values of $\Delta k_{0,SH} = 3.4 - 5.6$ and $\Delta k_{0,NH} = 5.9 - 8.9$. In the NH, due to the lack of a minimum of $g(\varphi)$ in the considered range, $f(TP1) + \sigma$ serves as an estimate for the upper limit.
Fig. S18 | The orography field $h_{or}$ of NH (top) and SH (bottom) mid-latitudes ($37.5^\circ$ - $57.5^\circ$ N, $30^\circ$S – $60^\circ$ S) employed for the calculation of the effective Forcing $F_{eff}$ [5].

Fig. S19 | Seasonal relative variability of the zonally averaged zonal wind in NH (JJA, blue) and SH (DJF, red) over a latitudinal range of $25^\circ$ – $77.5^\circ$. The relative variability in the NH exhibits highest values at latitudes at which waveguides for wave 6 -8 are situated ($30^\circ$N - $40^\circ$N). The SH in comparison shows its lowest values of relative variability in the regions where waveguides for waves 4, 5 are found ($35^\circ$S - $50^\circ$S).
**Fig. S20** Comparison between a 15 day low pass filter and a 15 day running mean applied to **a)** the zonally averaged zonal wind $U$ centered around the 31.06.2010 and **b)** the stationary wavenumber (Eq. [9]) as **a)** but for the meridional wavenumber. Both approaches deliver similar results.