Supplementary Material to “Super Clausius-Clapeyron scaling of extreme hourly convective precipitation and its relation to large-scale atmospheric conditions”

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Conceptual model

Our main assumption of the simplified model is that a fixed fraction of the latent heat release is converted to kinetic energy of the updraft $E$. We also assume that the dissipation of the $E$ is proportional to $E$ itself with a time scale $\tau$. In that case we can write a simplified equation:

$$\frac{\partial E}{\partial t} = f LP - E / \tau$$

Parameterizing the dissipation rate in this way is, we think, not unfair. The dissipation rate in the turbulent kinetic energy equation is usually taken proportional to $E^{1.5}$ divided by a length scale, which is $E$ divided by a time scale. The critical point here is assuming a fixed efficiency of the conversion of latent heat $LP$ into a production term of kinetic energy. The conversion should take place by the buoyancy term, and therefore this is likely dependent on the stability of the atmosphere. If the stability of the atmosphere increases according to a moist adiabat in a warming atmosphere then the additional latent heat release in the cloud does not lead to a warmer cloud compared to its environment, and it is likely that the efficiency $f$ to convert latent heat to kinetic energy goes down. In a more vertical uniform warming atmosphere our assumption however may not be unreasonable.

It is also useful to look at CAPE values to investigate how cloud vertical velocities may respond to warming. However, scaling of CAPE values is far from trivial. It is clear that the temperature of an updraft is related to the latent heating, and hence the moisture content, but the scaling of the integral of the updraft temperature excess is difficult as differences in the level of neutral buoyancy at the top of cloud also play a role: clouds developing from more humid conditions tend to grow deeper. Also, if the atmosphere in the reference state is close to the moist adiabat then CAPE values are likely to be small. In that case, CAPE increases could be substantially larger than expected since they are relative to the reference state with small CAPE values. So, relative changes in CAPE cannot be easily derived from simple arguments

In practice, however, we find for atmospheric conditions representative for the Netherlands a scaling of CAPE values close to $2\text{CC}$ for temperature perturbations uniform with height (see Figure 10 of Loriaux et al. 2013), which agrees with CC scaling for the updraft velocity. We
also remark that CAPE values are hardly affecting by temperature perturbations close to the moist adiabat (also shown in Figure 10 of that paper) clearly showing the importance of changes in vertical stability.
Evaluation hindcast in comparison with radiosonde

For evaluation of the hindcast results, we compared the output from the grid point closest to De Bilt with the radiosondes soundings at 12 UTC. The radiosonde profiles cover a slightly smaller time period as the 12 UTC radiosonde are not available for 2013 and later.

The temperature profiles from the model are very close to the radiosonde profiles with errors of typically 1-2 degrees, and a correlation of 0.98 or higher for the temperature at different atmospheric levels (Figure 1). For dew point temperature (measuring humidity) and relative humidity the differences between the model and the radiosonde is much larger on a case basis (Figure 1). We note however that the radiosonde profile does not necessarily reflect the mean conditions of the atmosphere very well, in particular for more convective situations where the radiosonde could well enter and leave clouds passing upward leading to substantial variability with height. We also note that the scatter plot based on ERA-interim is almost identical.

In the paper, however, we concentrate on the statistics averaged for a selection of cases. For these averages the model performs quite well. Figure 2 shows skew-T plots comparing the
RACMO2 hindcast prediction (+24 h) with the radiosonde for station De Bilt for two different selections of events. It is shown that the errors in the temperature profile are very small, mostly less than 0.3 degrees. The errors in the humidity profile are larger, but the difference between the model and the radiosonde is very similar for the different selections. It is known that a number of radiosonde sensors have a dry bias, so we cannot rule out that error is (partly) in the observations. We also note the bias directly derived from ERA-interim interpolated to De Bilt is almost identical. So, the model prediction of 24 ahead retains the quality as good as the ERA-interim. In addition to ERA-interim, the model provides much higher spatial and temporal resolution.

Figure 2. Skew-T plots showing radiosonde profiles (blue temperature, green humidity) versus hindcast results (red, temperature, orange humidity). Left, all dry events, middle all wet events, and right extreme events. Upper panel, dew point classification HR, lower panel dew point classification XH; see Table 1 of the main manuscript. Note that the temperature from the model and the radiosonde mostly overlap. The magenta dashed line is the temperature of a parcel lifted from the surface computed from the radiosonde.
Figure 3 shows the modelled temperature at 200m (in the atmospheric boundary layer) and at 5 km height in comparison with the radiosonde sounding, averaged across the different dew point temperature bins and intensity classifications. Temperature errors are generally small, within 0.5 ºC, but for a few classifications the errors in the boundary layer are slightly larger. Errors in relative humidity are larger typically within 5%, but occasionally up to almost 10%. Nevertheless, the main results of this figure corresponds well with Figure 12b,c of the main text.

Figure 3. Modelled (RACMO) and observed (radiosonde) temperature at 200 m and at 5 km, both at 12 UTC for location De Bilt (upper panels). Likewise, for relative humidity at 2 km and 5 km (lower panels). Averages are shown for the sample of events belonging to different intensity classification and the different dew point temperature bins. Observations are solid lines and square symbols, model results open circles and dashes lines.
Figure 4. Like Figure 3, but now for the temperature “excess” of a dry parcel lifted from 200 m to 3 and 5 km (essentially a measure of the instability between 200 m and 3 and 5 km height).

Figure 5. Modelled lifted dry temperature excess from 200 m to 3 (left) and 5 km (right), but now for the timing of 3 hours before the peak intensity of an event and co-allocated at the station of occurrence of the peak intensity (compare to Figure 4, derived for 12 UTC and station De Bilt and a slightly smaller data set).
Figure 4 shows a measure of dry atmospheric stability between 200 m, and 3 and 5 km height, comparable to Figure 12d,e of the main text. Here we used a dry parcel lifted from 200 m (instead of the surface) in order to have a clean comparison between model results and observations, noting that the surface parcel is lifted using observed surface temperature (and humidity). In comparison we show a reproduction of Figure 12d,e of the main text, but now using a parcel starting at 200 m (Figure 5). The latter figure differs with Figure 4 with respect to the timing and the location. Figure 4 is based on 12 UTC observations at station De Bilt, whereas Figure 5 is based on the timing of 3 hours before the event and collocated with the station where the peak intensity is observed (like in the main text). By comparing Figure 4 with Figure 5, however, it is clear that results are quite robust.

Figure 6. Comparison between observed rainfall accumulation of 24-h (from 12 hours before the event until 12 hours after), averaged over all AWS stations in the Netherlands. Averages are shown for the sample of events belonging to different intensity classification and the different dew point temperature bins. Observations are solid lines and square symbols, model results open circles and dashes lines.

Figure 6 shows a comparison between modelled and observed rainfall averaged over all weather stations and accumulated from 12 hours before until 12 hours after the event. Results are for different intensity classification and dew point bins (compare to figure 11c of the main
text). For these sample mean and aggregated statistics the model reproduces the observations to a very good degree. Given the similarity between observed accumulated precipitation and moisture convergence related to omega (Figure 11c and 11d of the main text) this gives us confidence that the modelled omega field is realistic in the sample mean aggregated sense.
Convective available potential energy (CAPE) and shear.

Here we present CAPE and horizontal wind shear between 5 km and 10 m, which are often used to predict the severity of storms (Brooks et al. 2003; Craven and Brooks 2004). CAPE again shows the strong increase with surface humidity, but for a given dew point temperature are rather weak distinction between the intensity classifications (Figure 7). The wind shear hardly shows a difference between weak and strong events and also almost no dependency on the dew point temperature. Apparently, wind shear is not a good predictor of hourly precipitation. Storms organisation may well increase with wind shear and lead to higher precipitation amounts from showers, however increasing propagation velocities of the convective showers may lead to reduced precipitation amounts at surface stations.

Figure 7. Convective available potential energy (CAPE) and wind shear (vector difference between 10m and 5 km winds) at 3 hours before the event as a function of near surface dew point temperature, and for the different intensity classifications.
Quasi-geostrophic omega equation

Here, we illustrate with the Quasi-Geostrophic omega equation that latent heat release associated with convection could lead to a response in vertical velocity. The argument only gives an order of magnitude estimate and should not be considered to be accurate in a quantitative sense.

Starting point is a simplified form of the quasi-geostrophic omega equation as given in Equation 5 from Nie and Sobel (2016):

\[
\frac{\partial}{\partial p} \omega - \sigma \left( \frac{k}{f_0} \right)^2 \omega = -\frac{1}{f_0} \frac{\partial}{\partial p} \text{Adv}_v + \frac{R}{p} \left( \frac{k}{f_0} \right)^2 \text{Adv}_T + \frac{R}{p} \left( \frac{k}{f_0} \right)^2 Q
\]

where \( k = 2\pi/L \), with \( L \) a characteristic horizontal length scale for a disturbance. Here \( Q \) is diabatic heating rate. For all other terms see Nie and Sobel (2016), page 1103.

We used a typical diabatic heating term associated with a selection of intense precipitation events, which is shown in Figure 4b of Loriaux et al. (2016). The selection of events in that study have a mean peak intensity of 8 mm hour\(^{-1}\), and range between 4 and 79 mm hour\(^{-1}\), which corresponds roughly to events from the heavy to extreme intensity classification at medium to high humidity (see Figure 4 and Table 1). It follows approximately a sine dependency with height in pressure coordinates, peaking halfway the troposphere at approximately 600 hPa, and gradually decreasing to zero at 200-300 hPa. Peak values are approximately 5 K per day, or 5 \( \times 10^5 \) K s\(^{-1}\). The selection of events used here is close to our extreme selection based on the HR humidity range.
Based on this, and following the same methodology as in Nie and Sobel (2016), we assume omega to have the following vertical dependency:

$$\omega = \tilde{\omega} \sin(mp)$$

with $m = \pi/p^*$, with $p^*$ the depth of the troposphere, or approximately 800 hPa.

Filling this in, we arrive at

$$- \left[ m^2 + \sigma \left( \frac{k}{f_0} \right)^2 \right] \omega = - \frac{1}{f_0} \partial_p A_c + \frac{R}{p} \left( \frac{k}{f_0} \right)^2 A_T + \frac{R}{p} \left( \frac{k}{f_0} \right)^2 Q$$

This is now a linear equation, and we can compute the contribution of the diabatic heating term to omega.

Assuming a horizontal disturbance with $L = 600$ km, and taking $\sigma \approx 2 \times 10^{-6}$ m$^2$ Pa$^{-2}$ s$^{-2}$ (page 1104; Nie and Sobel 2016), and $f \approx 10^{-4}$ s$^{-1}$, so that $k/f_0 \approx 10^{-1}$ s$^{-1}$, and $m \approx 0.4 \times 10^{-4}$ Pa$^{-2}$, so that the term between square brackets on the left hand side is $\approx 2 \times 10^{8}$ Pa$^{-2}$. Finally, using $R = 287$ J kg$^{-1}$ K$^{-1}$, and considering the middle troposphere so that $p \approx 500$ hPa, we get that $\omega \approx 10^{-1}$ Pa s$^{-1}$, which is $\approx 4$ hPa hour$^{-1}$. 