

1 **A new method of comparing forcing agents in climate models**

2 **(Supplemental Online Material)**

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## ABSTRACT

<sup>17</sup> This document contains one section of supplemental online material and one  
<sup>18</sup> table, all dealing with transitivity of the relationships described in Table 1 of  
<sup>19</sup> the main text.

## 20 1. Transitivity

21 Transitivity as used here is taken from the mathematical concept of the same name and is an  
22 assessment of how well the regressed relationships can be combined to explain each other. As a  
23 broad example, if  $2x\text{CO}_2$  is equivalent to  $X\%$  solar, and  $X\%$  solar is equivalent to  $Y\text{CH}_4$ , then is  
24  $2x\text{CO}_2$  equivalent to  $Y\text{CH}_4$  according to our method? More specifically, if one knows Equations  
25 8 and 10 (See Table 1), then how accurate is the result of combining these two equations to yield  
26 Equations 12 and 13?

27 In the absence of noise and any remaining nonlinearities, transitivity should hold perfectly. How-  
28 ever, because we are recovering the regressed relationships in the presence of noise, transitivity  
29 may be challenging to recover. Additionally, as was discussed in Section 4a, relationships between  
30 certain combinations of forcing agents may be more difficult to recover than others. Our exercise  
31 here in testing transitivity is not to test whether transitivity is a fact, but rather to provide a mea-  
32 sure of this method's ability to recover accurate relationships between the forcing agents, including  
33 both the scaling relationships and the coefficients obtained in the regressions. Section 4a gives ex-  
34 amples of three sources of uncertainty in the explicit feedback method that could contribute to  
35 challenges in recovering these coefficients:

- 36 1. Large relative changes in the amount of a particular forcing agent are required to obtain the  
37 comparison relationship
- 38 2. Inabilities to achieve required levels of forcing, such as an inability to cause an offset (e.g.,  
39 reduction in  $\text{CH}_4$  to compensate for an increase in  $S_0$ ) or reverse saturation (e.g., very low  
40 concentrations of  $\text{CH}_4$ )
- 41 3. Nonlinearities that are not related to (global mean) temperature-induced feedbacks

42 Here we assess the ability of our method to recover transitivity by substituting the equations  
43 in Table 1 into each other to recover other relationships in the presence of noise. We also invert  
44 the equations in Table 1. Performing all possible substitutions and inversions yields the results in  
45 the table below. Tabulation of the aggregate percent error in slope associated with each equation  
46 used in the substitution (Column 3 in the table below) yields a determination of which regressed  
47 relationships are associated with the least amount of error. When only tabulating substitutions  
48 and not inversions, the error due to Equations 8 and 10 is substantially lower than the error of  
49 Equations 12-15. Equations 12-15 can then be derived from Equations 8 and 10 via substitution  
50 or inversion. When including inversions in the tabulation, Equation 10 still has the lowest error  
51 of all equations. Of the three forcing agents in this study, using the feedback loop to change  
52 total solar irradiance will result in the most accurate comparison. This follows from the discussion  
53 elucidated in Section 4a; for example, meeting any goal of comparison with  $S_0$  requires only small  
54 adjustments, whereas compensating adjustments in  $CH_4$  often require very large relative changes,  
55 some of which are not even achievable.

56 As was described in Section 4a, the regressed relationships, and hence transitivity, are only valid  
57 within a certain range of forcings. For example, when offsetting  $CH_4$  changes by modifying  $CO_2$ ,  
58 very low values of methane cannot be captured by the regression relationship, so transitivity would  
59 not be expected for these low values.

60 **LIST OF TABLES**

61 **Table 1.** Comparison between regressed relationships in GISS ModelE2 (Equations 8,  
62 10, 12-15) and recovered equations from substitution or inversion. First two  
63 columns describe the independent and dependent variables in the equation.  
64 Third column indicates which two equations were combined or which equation  
65 was inverted such that the result can be compared with an equation obtained  
66 from regression (fourth column). Columns 5 and 8 show the slope and intercept  
67 (respectively) of the relation obtained by substitution. Columns 6 and 9  
68 show the slope and intercept of the original relation. Column 7 shows the per-  
69 cent error, calculated as (slope minus original slope)/(original slope). % error  
70 is not given for intercepts, as such calculations would be dependent upon a cho-  
71 sen baseline and hence arbitrary. Slope and intercept values are rounded to four  
72 decimal places, and percent error is rounded to one decimal place. . . . . 6

Indep. Var.	Dep. Var.	Eqns.	Orig. Eqn.	Slope	Orig. Slope	Error (%)	Int.	Orig. Int.
$\log_2(\text{XCO}_2)$	$\text{XS}_0$	10,12	8	-0.0219	-0.0208	5.0	0.0097	-0.0010
$\sqrt{\text{CH}_4}$	$\text{XS}_0$	8,13	10	-0.0068	-0.0073	6.6	0.0032	0.0051
$\log_2(\text{XCO}_2)$	$\sqrt{\text{CH}_4}$	13,15	12	-2.5048	-2.9866	16.1	241.1501	0.6210
$\sqrt{\text{CH}_4}$	$\log_2(\text{XCO}_2)$	12,14	13	-0.2718	-0.3284	17.2	74.4295	0.2037
$\text{XS}_0$	$\log_2(\text{XCO}_2)$	10,14	14	-39.4923	-37.1243	6.4	0.4570	-0.0103
$\text{XS}_0$	$\sqrt{\text{CH}_4}$	8,15	15	-110.8766	-120.2520	7.8	0.5904	0.7713
$\log_2(\text{XCO}_2)$	$\text{XS}_0$	12,15	8	-0.0248	-0.0208	19.2	0.0116	-0.0010
$\sqrt{\text{CH}_4}$	$\text{XS}_0$	13,15	10	-0.0088	-0.0073	20.8	0.0052	0.0051
$\log_2(\text{XCO}_2)$	$\sqrt{\text{CH}_4}$	8,10	12	-2.8451	-2.9866	4.7	273.7377	0.6210
$\sqrt{\text{CH}_4}$	$\log_2(\text{XCO}_2)$	8,10	13	-0.3515	-0.3284	7.0	96.2146	0.2037
$\text{XS}_0$	$\log_2(\text{XCO}_2)$	12,15	14	-40.2634	-37.1243	8.5	0.4662	-0.0103
$\text{XS}_0$	$\sqrt{\text{CH}_4}$	13,15	15	-113.0415	-120.2520	6.0	0.5889	0.7713
$\log_2(\text{XCO}_2)$	$\text{XS}_0$	14	8	-0.0269	-0.0208	29.3	-0.0003	-0.0010
$\sqrt{\text{CH}_4}$	$\text{XS}_0$	15	10	-0.0083	-0.0073	13.6	0.0064	0.0051
$\log_2(\text{XCO}_2)$	$\sqrt{\text{CH}_4}$	13	12	-3.0449	-2.9866	0.2	0.6201	0.6210
$\sqrt{\text{CH}_4}$	$\log_2(\text{XCO}_2)$	12	13	-0.3348	-0.3284	0.2	0.2079	0.2037
$\text{XS}_0$	$\log_2(\text{XCO}_2)$	8	14	-48.0087	-37.1243	29.3	-0.0050	-0.0103
$\text{XS}_0$	$\sqrt{\text{CH}_4}$	10	15	-136.5883	-120.2520	13.6	0.7032	0.77133

73 TABLE 1. Comparison between regressed relationships in GISS ModelE2 (Equations 8, 10, 12-15) and re-  
74 covered equations from substitution or inversion. First two columns describe the independent and dependent  
75 variables in the equation. Third column indicates which two equations were combined or which equation was in-  
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81 percent error is rounded to one decimal place.