Supplementary Material

Spatial and temporal characterization of sea surface temperature response to tropical cyclones

1 The simple mixed layer model solution with a time-varying wind stress

If we assume the wind stress changes periodically in time as

\[ \tau = (\tau_0 \sin(\omega t), \tau_0 \cos(\omega t)) \],

(1.1)

we can get the solutions to Eq. (A1) in the appendix of the manuscript as

\[ h_u = \frac{\tau_0}{\rho(\omega - f)} \left( + \cos(ft) - \cos(\omega t) \right), \]

\[ h_v = \frac{\tau_0}{\rho(\omega - f)} \left( - \sin(ft) + \sin(\omega t) \right). \]

(1.2)

Then the mixing depth, which oscillates in time, is

\[ h = \sqrt{h_{cml}^2 + \sqrt{h_{cml}^4 + \frac{16Ri\tau_0^2}{N^2(\omega - f)^2 \rho^2} \left(1 - \cos((\omega - f)t)\right)}} \].

(1.3)

The maximum mixing depth

\[ h_{max} = \sqrt{h_{cml}^2 + \sqrt{h_{cml}^4 + \frac{32Ri\tau_0^2}{N^2(\omega - f)^2 \rho^2}}} \].

(1.4)

is obtained for \( t \geq \pi/|\omega - f| \). It is evident that both the magnitude and the sign of \( \omega \) can significantly affect the wind-induced mixing depth.
Using this maximum mixing depth and Eq. (A8) in the appendix of the manuscript, we get the maximum SST change $T_{\text{max}}$, which is shown in Fig. S1 as function of wind speed for a specific choice of parameters, listed below.

In the case $\omega = f$, a resonant effect leads to a mixed layer depth that, for limited time $t$, grows as

$$ h \simeq \sqrt{h_{\text{cml}}^2 + \left( h_{\text{cml}}^4 + \frac{8\text{Ri}\tau_0^2 t^2}{N^2 \rho^2} \right)} . \quad (1.5) $$

For this resonant condition, we plot the value of $T_{\text{max}}$ corresponding to the mixed layer depth at several different times $t$ (see Fig. S2). The SST anomaly has a quasi-linear dependence on wind speed.

The parameters in the above equations used to produce the Figs. S1 and S2 are: $h_{\text{cml}} = 25$ m, $\text{Ri} = 1$, $g = 9.8 \text{ m s}^{-2}$, $\alpha = 2 \times 10^{-4} \text{ K}^{-1}$, $\Gamma = 0.06 \text{ K m}^{-1}$, $f = 6 \times 10^{-5} \text{ s}^{-1}$, air density $\rho_a = 1.25 \text{ kg m}^{-3}$, water density $\rho_o = 1.025 \times 10^3 \text{ kg m}^{-3}$, and drag coefficient $C_d = 2 \times 10^{-3}$. $N^2$ and wind stress $\tau_0$ are obtained via $N^2 = g \alpha \Gamma$ and $\tau_0 = \rho_a C_d V_s^2$ (where $V_s$ is the maximum sustained wind speed of a tropical cyclone), respectively.
Figure S1: Maximum SSTA as a function of the tropical cyclone intensity obtained from the simple mixed layer model. Solid line is same as the thick solid curve shown in Fig. 8a of the manuscript. Dashed line shows the situation with a time-varying wind stress and $\omega = 2.62 \times 10^{-5}$ s; dash-dot line is for $\omega = -2.62 \times 10^{-5}$ s.
Figure S2: SSTA as a function of the tropical cyclone intensity obtained from the simple mixed layer model with time-varying wind stress and $\omega = f$. Black curve is same as the thick solid curve shown in Fig. 8a of the manuscript. Curves in other colors are for $t = 1/4$ day (blue), $t = 1/2$ day (green), $t = 3/4$ day (red), and $t = 1$ day (magenta).
Figure S3: The SSTA simulated by the simple mixed layer at the radius of maximum wind speed (magenta curve), averaged within a 100-km radius (red curve), averaged within a 200-km radius (black curve), and averaged within a 300-km radius (blue curve) as a function of TC intensity that is expressed as the maximum surface wind speed.