Supplemental Material

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Calculating State Dependent Noise in a Linear Inverse Model Framework

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1 Linearity of the time resolved dynamics

As a test of the linearity of the resolved dynamics we use a variation of a "τ test" (Penland and Sardeshmukh 1995; Newman and Sardeshmukh, 2017). For this purpose we use matrix $\mathbf{M}$ (equation 29 main text) calculated using contemporary and 6 days lag covariance matrices, to generate the predicted LIM and CAM-LIM lag covariances at remaining lags. This is calculated as follows,

$$\mathbf{C}_\tau = exp(\mathbf{M}\tau)\mathbf{C}_0. \tag{1}$$

The result is shown in figure S1. Notice that in this particular case lag covariances and lag correlations coincide. Here C_{11}^τ denote T_a autocorrelation function, C_{22}^τ T_o autocorrelation function, C_{12}^τ denotes $< T_a(\tau)T_o(0) >$, and C_{21}^τ denotes $< T_o(\tau)T_a(0) >$. With a small error for T_a autocorrelations for lag of 8 days or more, this figure shows that the lag correlation functions can be well described by linear dynamics. Thus a linear system driven by state dependent noise provides a good paradigm for the local ocean-atmosphere coupled variability at OWS P.

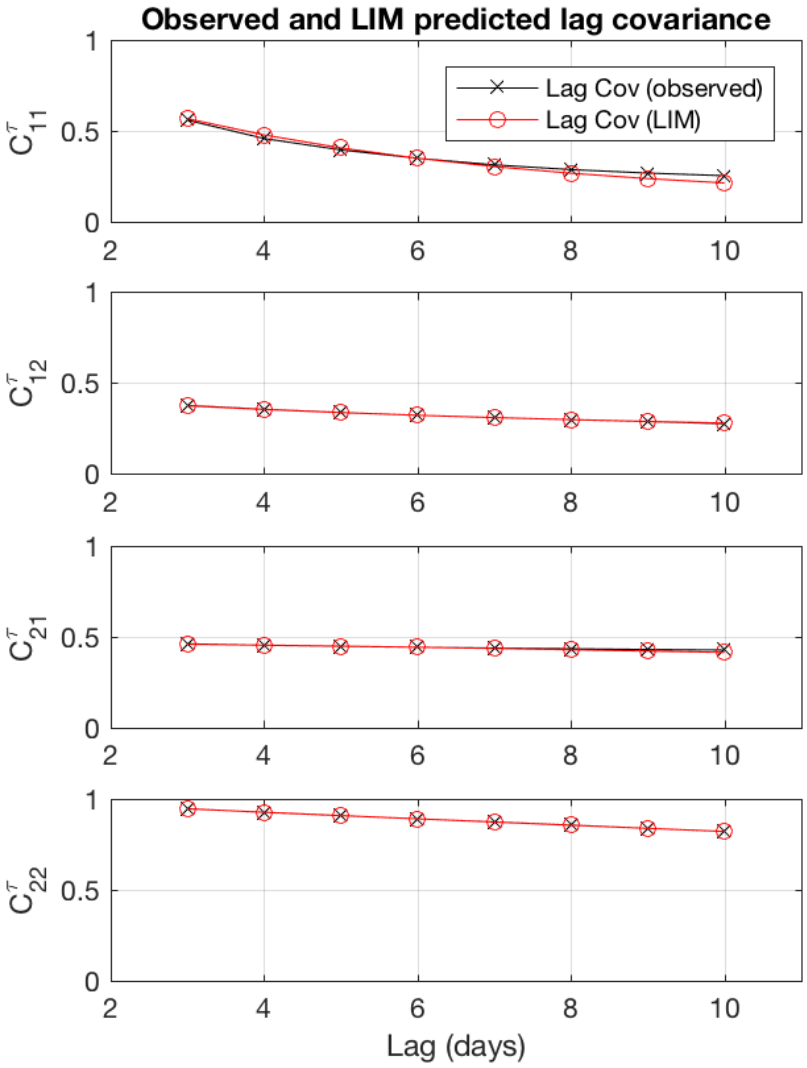

Figure S1: Observed lag covariance $C_\tau =< T(\tau)T(0)^T >$ (where $T = [T_A T_O]^T$), and standard LIM and CAM-LIM predicted $C_\tau = exp(M\tau)C_0$, where C_0 is the observed contemporary covariance. This notation is only used in this figure.

2 Parameter variations as a function of lag.

Figure S2 shows the variations in E_i, and G_i $(i = 1, 2)$ calculated values as a function of lag. Values stay within $\sim 10\%$ of the quoted values (equation 29) for lags 3 to 10 days.

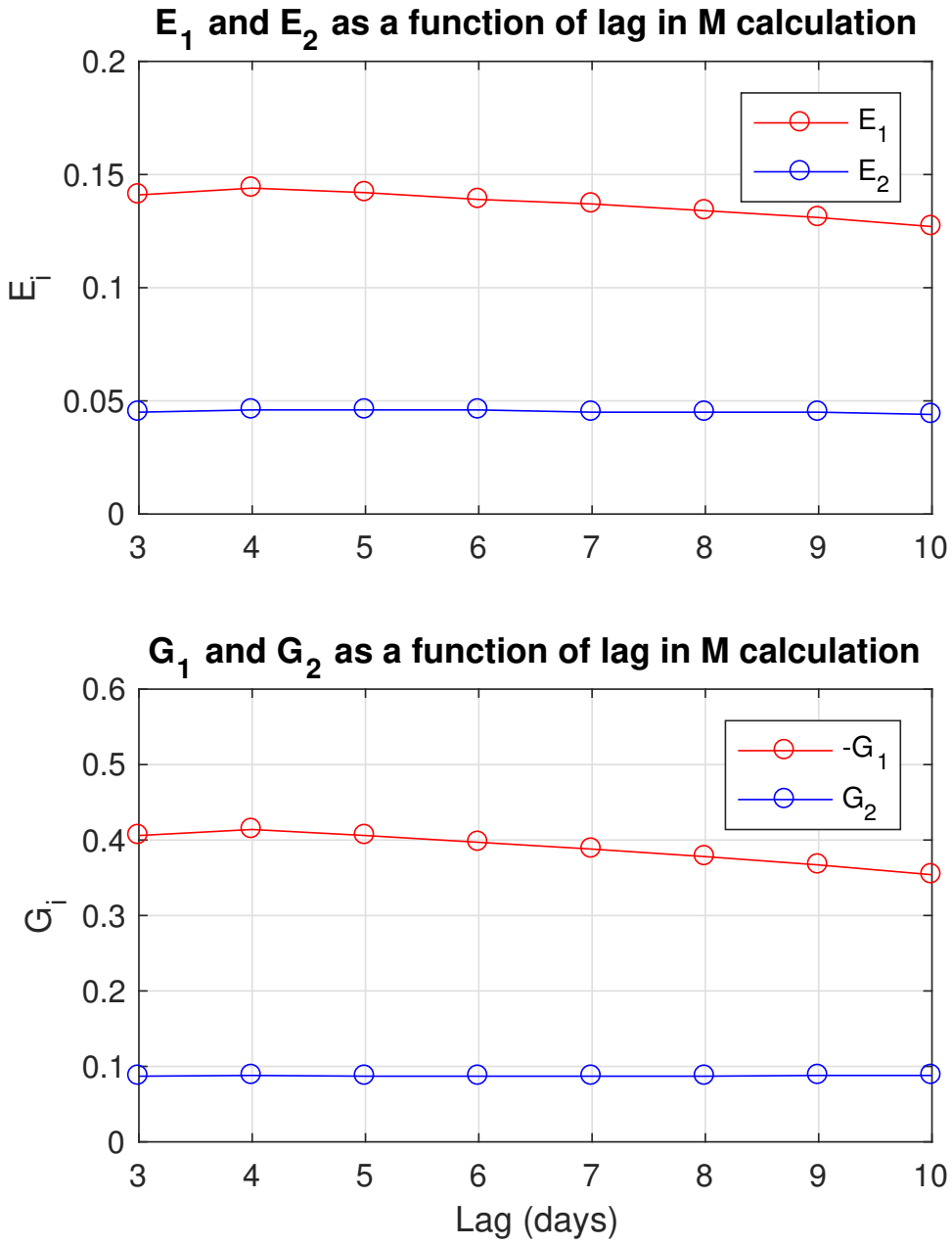

Figure S2: **Top** Value of E_1, E_2 as a function of τ used to calculate **M**. **Bottom** Value of E_1, E_2 as a function of τ used to calculate **M**. Notice that the G_1 values are multiplied by -1.

3 Figure 3 on a vertical logarithmic scale.

This figure shows figures 3c to 3f but using a logarithmic y axis, corresponding to figures S3a to S3d, to highlight differences between distributions at the tails.

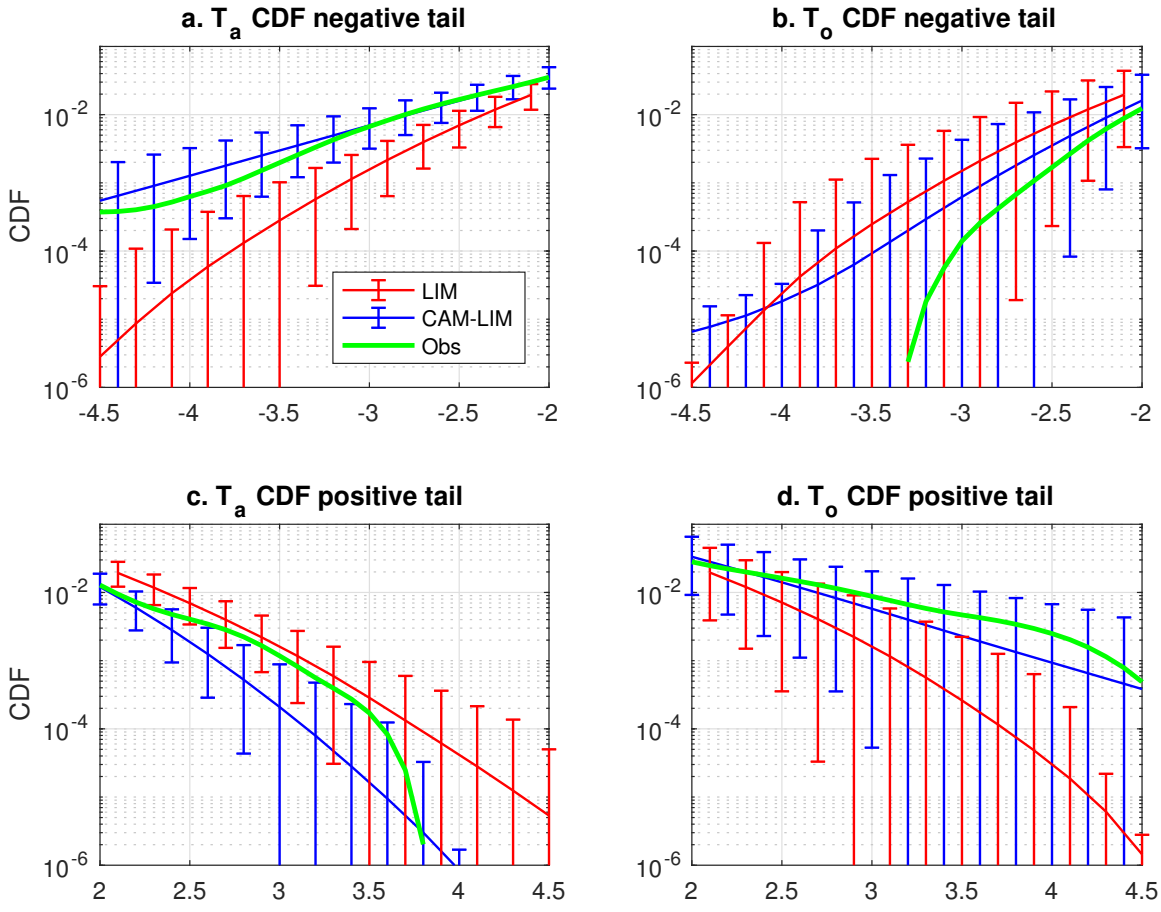

Figure S3: **a-b-c-d** Same information as figure 3c-d-e-f, but using a logarithmic y axis.