



**AMS**  
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## Supplemental Material

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# Supplemental Material

## Calculating State Dependent Noise in a Linear Inverse

### Model Framework

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#### 1 Linearity of the time resolved dynamics

As a test of the linearity of the resolved dynamics we use a variation of a “ $\tau$  test” (Penland and Sardeshmukh 1995; Newman and Sardeshmukh, 2017). For this purpose we use matrix  $\mathbf{M}$  (equation 29 main text) calculated using contemporary and 6 days lag covariance matrices, to generate the predicted LIM and CAM-LIM lag covariances at remaining lags. This is calculated as follows,

$$\mathbf{C}_\tau = \exp(\mathbf{M}\tau)\mathbf{C}_0. \quad (1)$$

The result is shown in figure S1. Notice that in this particular case lag covariances and lag correlations coincide. Here  $C_{11}^\tau$  denote  $T_a$  autocorrelation function,  $C_{22}^\tau$   $T_o$  autocorrelation function,  $C_{12}^\tau$  denotes  $\langle T_a(\tau)T_o(0) \rangle$ , and  $C_{21}^\tau$  denotes  $\langle T_o(\tau)T_a(0) \rangle$ . With a small error for  $T_a$  autocorrelations for lag of 8 days or more, this figure shows that the lag correlation functions can be well described by linear dynamics. Thus a linear system driven by state dependent noise provides a good paradigm for the local ocean-atmosphere coupled variability at OWS P.

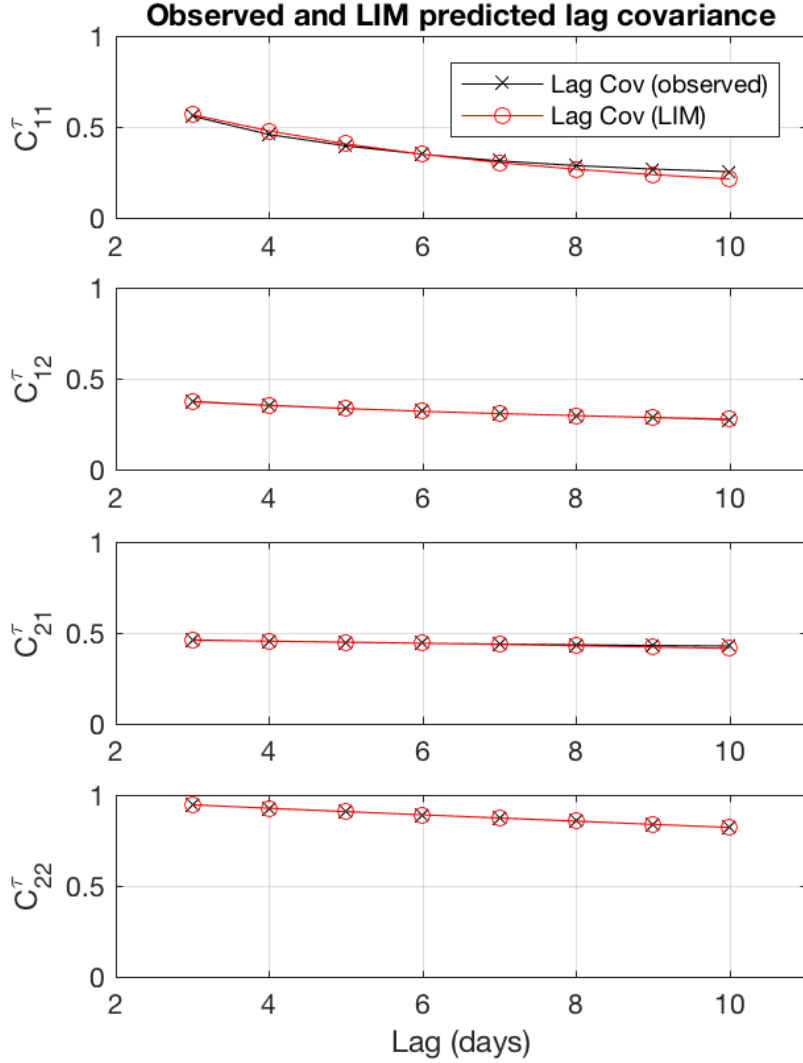


Figure S1: Observed lag covariance  $C_{\tau} = \langle T(\tau)T(0)^T \rangle$  (where  $T = [T_A T_O]^T$ ), and standard LIM and CAM-LIM predicted  $C_{\tau} = \exp(M\tau)C_0$ , where  $C_0$  is the observed contemporary covariance. This notation is only used in this figure.

## 2 Parameter variations as a function of lag.

Figure S2 shows the variations in  $E_i$ , and  $G_i$  ( $i = 1, 2$ ) calculated values as a function of lag. Values stay within  $\sim 10\%$  of the quoted values (equation 29) for lags 3 to 10 days.

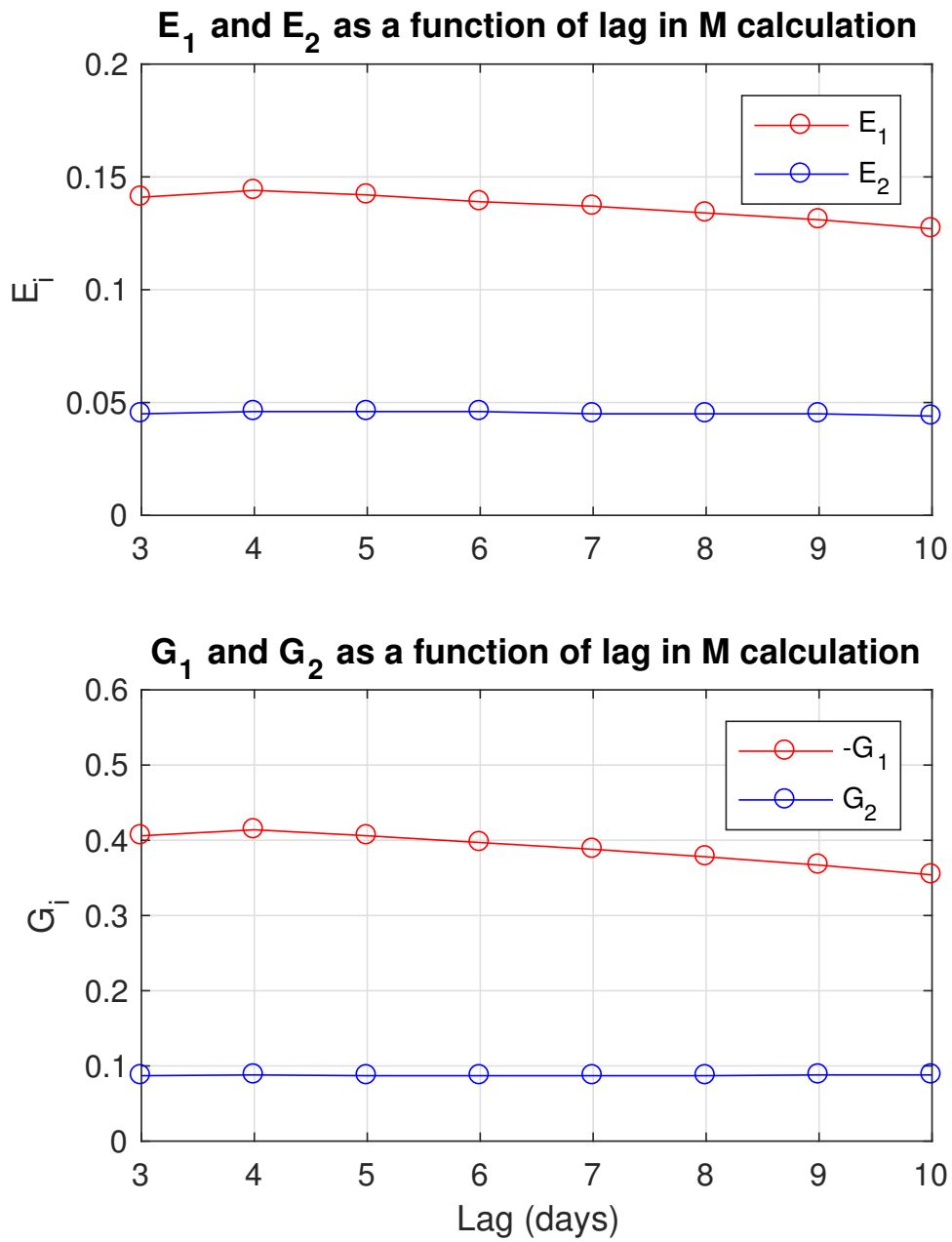


Figure S2: **Top** Value of  $E_1$ ,  $E_2$  as a function of  $\tau$  used to calculate **M**. **Bottom** Value of  $E_1$ ,  $E_2$  as a function of  $\tau$  used to calculate **M**. Notice that the  $G_1$  values are multiplied by -1.

### 3 Figure 3 on a vertical logarithmic scale.

This figure shows figures 3c to 3f but using a logarithmic y axis, corresponding to figures S3a to S3d, to highlight differences between distributions at the tails.

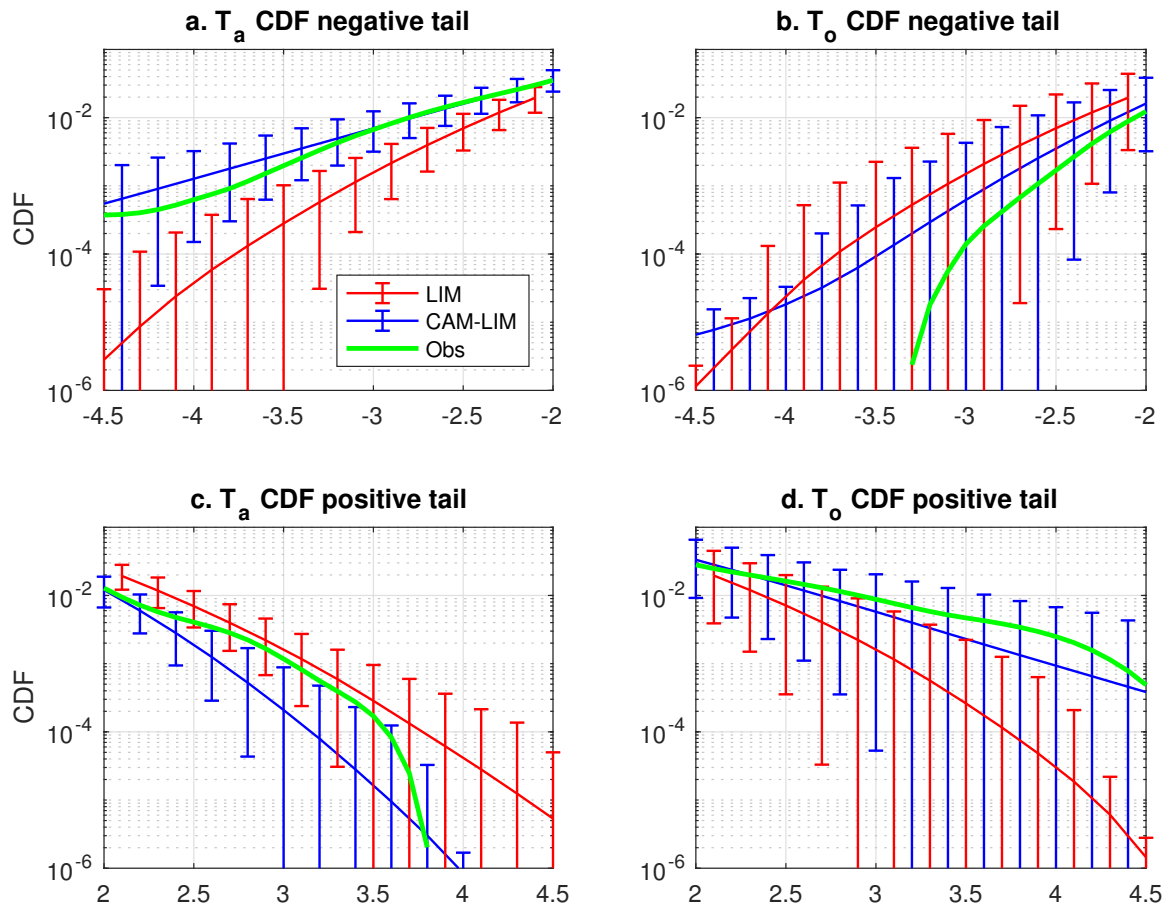


Figure S3: **a-b-c-d** Same information as figure 3c-d-e-f, but using a logarithmic y axis.